Seismic Trace Interpolation using Adaptive Prediction Filters

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Summary

A new seismic trace interpolation method based on adaptive prediction filters (PF) is introduced. In order to estimate local PFs, the contribution of adjacent windows of data to the process of local PF estimation is reduced via the introduction of a forgetting factor. Using a Recursive Least Square (RLS) algorithm, the problem of estimating PFs for each window of data is avoided and the local PF for a current data window is updated from available local PFs computed from adjacent windows. Finally, local PFs computed at low frequencies are used to interpolate data at high frequencies using Spitz reconstruction algorithm (1991). We provide synthetic and real data examples to examine the performance of the proposed adaptive PF interpolation algorithm.

Introduction

Spitz (1991) introduced a seismic trace interpolation method that uses Prediction Filters (PF) in the f-x domain. This method is based on the fact that linear events in the time-space domain can be efficiently modeled via PFs in the frequency-space domain. Spitz (1991) also showed that PFs computed at low frequencies can be used to interpolate spatial aliased data at high frequencies. Similar algorithms were proposed by Porsani (1991) and Gulunay (2003).

In an attempt to use PFs for the reconstruction of irregularly sampled data, Naghizadeh and Sacchi (2007) introduced the Multi-Step Auto-Regressive (MSAR) reconstruction method. In the MSAR method, first the low frequency portion of data is reconstructed using a Fourier reconstruction algorithm (Liu and Sacchi, 2004). Next, PFs for all frequencies are extracted from the already regularized low frequency portion of data by applying multi-step autoregressive operators.

In general, we can say that a main limitation of f-x prediction filters (Spitz, 1991; Porsani 1999, Naguizadeh and Sacchi, 2007) is that they are built under the assumption of constant dip. In order to relax the aforementioned assumption, fx interpolation is often run on small spatial windows. Here we tackle this problem by introducing the concept of adaptive PFs. We aim to compute the adaptive PFs using a recursive scheme that does not require estimation of the PF and reconstruction for each window

Theory

Consider a length-*N* discrete signal $\mathbf{x} = \{x_1, x_2, x_3, \ldots, x_N\}$. Assuming constant length M for the PFs, we aim to find the best local PF for each window of data with length equal to $M+1$. The adaptive PF estimation using recursive least squares can be found in Honig and Messerschmidt (1984) as well as Haykin (2002) and can be summarized as follows. We first define $\mathbf{u}(i) = [x_{i+M-1}, x_{i+M-2}, \ldots, x_i]^T$ and $d(i) = x_{i+M}$, and the adaptive PF for the n th window of the data $\mathbf{p}(n)$. The following recursion is used to update the predictor filter:

$$\begin{aligned}
\mathbf{w}(n) &= \frac{\lambda^{-1}\mathbf{R}(n-1)\mathbf{u}(n)}{1+\lambda^{-1}\mathbf{u}(n)^T\mathbf{R}(n-1)\mathbf{u}(n)} \\
\alpha(n) &= d(n) - \mathbf{u}(n)^T\mathbf{p}(n-1) \\
\mathbf{p}(n) &= \mathbf{p}(n-1) + \mathbf{w}(n)\alpha(n) \\
\mathbf{R}(n) &= \lambda^{-1}\mathbf{R}(n-1) - \lambda^{-1}\mathbf{w}(n)\mathbf{u}(n)^T\mathbf{R}(n-1)
\end{aligned} \tag{1}$$

where, $0 < \lambda \le 1$ is the forgetting factor which is used to reduce the effects of adjacent windows on the estimation of the current PF. In order to initiate the recursive estimation algorithm (1), the values of $\mathbf{R}(1)$ and $\mathbf{p}(1)$ associated to the first window of data are required. The algorithm is initialized with the following quantities: $c(i) = x_i^*$ and $\mathbf{v}(i) = [x_{i+1}^*, x_{i+2}^*, \ldots, x_{i+M}^*]^T$,

$$\mathbf{R}(1) = \left(\sum_{i=1}^{n} \lambda^{i-1}\mathbf{v}(i)\mathbf{v}(i)^T\right)^{-1} \tag{2}$$

$$\mathbf{p}(1) = \mathbf{R}(1)\left(\sum_{i=1}^{n} \lambda^{i-1}\mathbf{v}(i)c(i)\right) \tag{3}$$

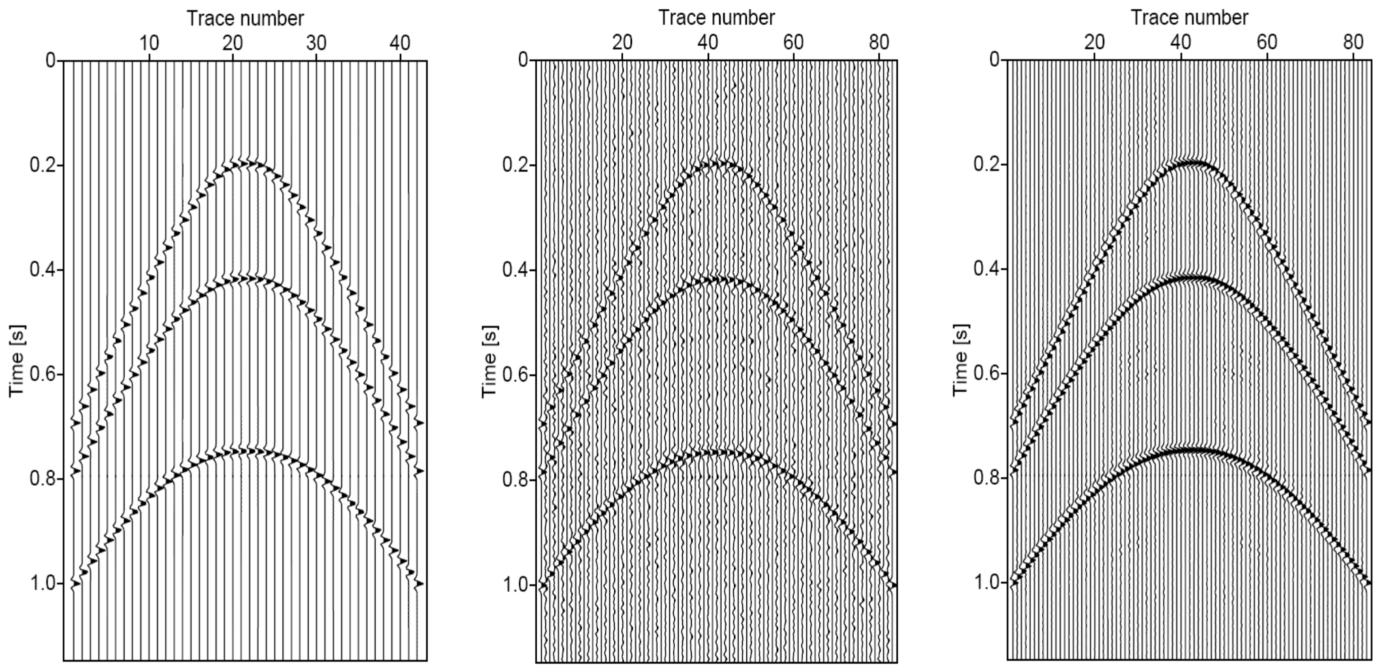
where, * represents the complex conjugate and T stands for the transpose. Notice that, equation (2) is the only inversion required for the adaptive PF estimation.

In order to use the adaptive PFs for interpolation purposes, we first transform the data into the f-x domain. Then, the spatial samples of each single frequency are considered as the signal $\mathbf{x}$. The adaptive PFs computed at the frequency $f/2$ and later used to interpolate the data samples at frequency f. Each local PF interpolates in between the samples of its associated window. An augmented matrix of linear system of equations is built from all the local PFs and it is solved to find the interpolated samples. The latter follows very closely the approach proposed by Spitz (1991). It is important to stress, however, that the reconstruction uses local filters and not a global one like in the procedure introduced by Spitz (1991).

Examples

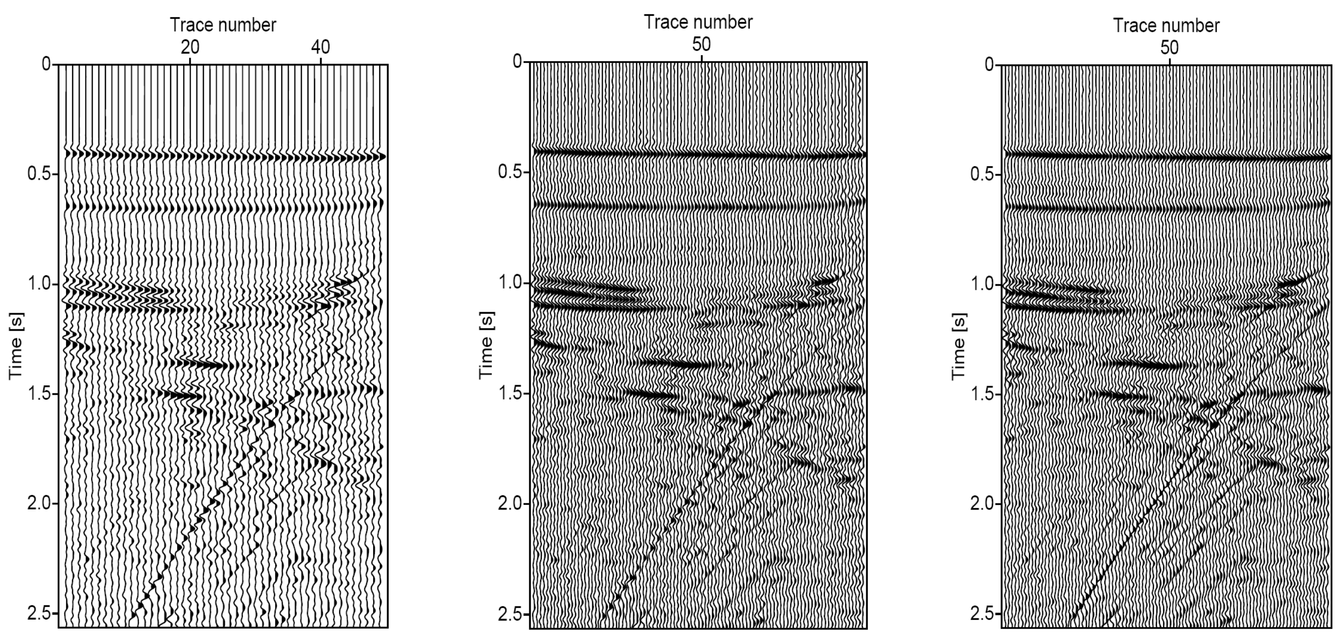
In our first example we have created a synthetic section with hyperbolic events (Figure 1a). We interpolated the original section using local PFs of length *M=3* and forgetting factors $\lambda = 1$ (Figure 1b) and $\lambda = 0.25$ (Figure 1c). Choosing the value $\lambda = 1$ means that all processed windows will contribute to the estimation of local PF and the interpolated data will be similar to the one we would have obtained using the classical (stationary) f-x interpolation proposed by Spitz (1991). By reducing the value of λ, the contribution from the neighbor windows are decreased exponentially and the estimated local PFs represent the local slope of the events.

Figure 2a shows a common offset gather of the data set from the Gulf of Mexico containing diffractions. The data are interpolated using adaptive PFs with length equal to 4 and forgetting factors $\lambda = 1$ (Figure 2b) and $\lambda = 0.45$ (Figure 2c). The interpolation using $\lambda = 1$ struggles to interpolate every dip in the data. The interpolation using $\lambda = 0.45$, on the other hand, shows an important improvement in dip adaptability.



a) b) c)

Figure 1: a) Original data. b) Interpolated data using $\lambda = 1$. c) Interpolated data using $\lambda = 0.25$.



a) b) c)

Figure 2: a) Original common offset gather from Gulf of Mexico. b) Interpolated using $\lambda = 1$. c) Interpolated using $\lambda = 0.45$.

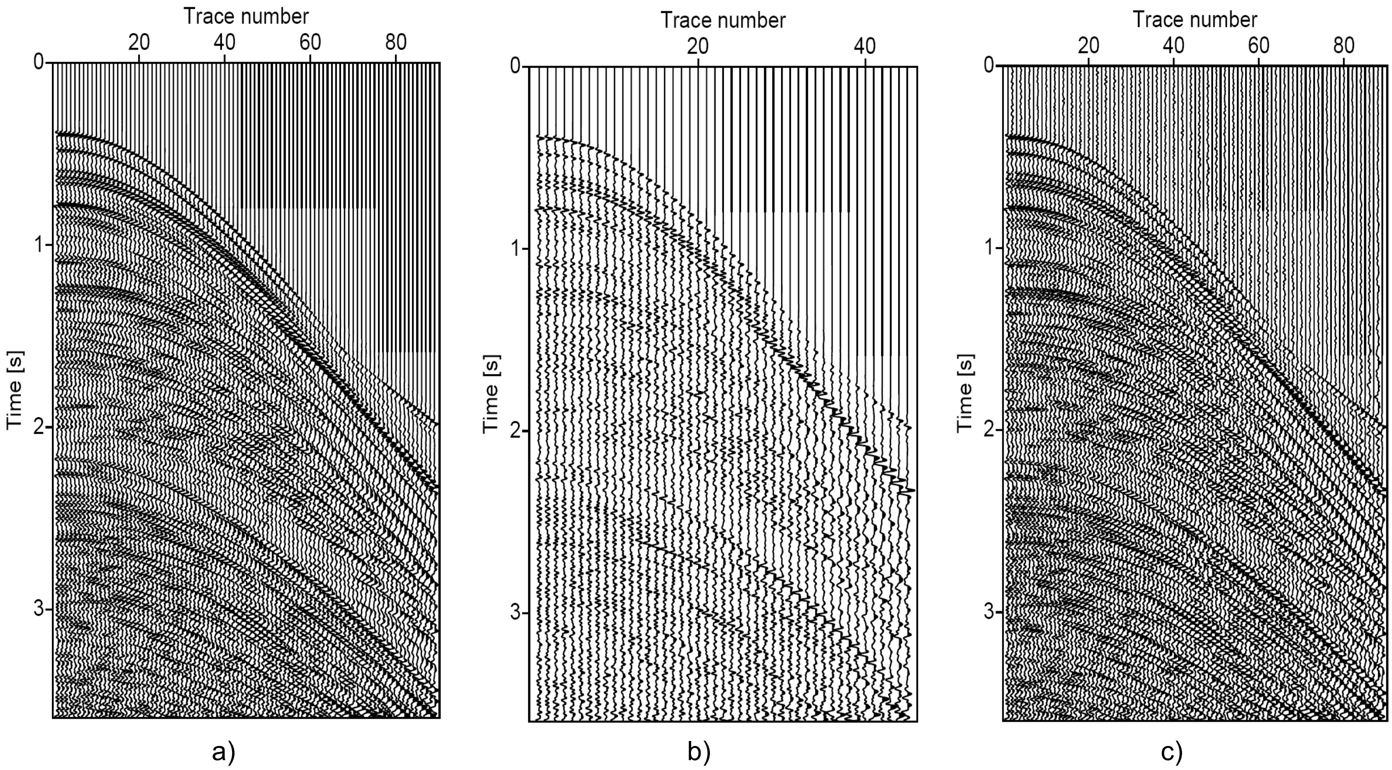


Figure 3: a) Original shot gather from Gulf of Mexico; b) Decimated shot gather; c) Interpolated data using $\lambda = 0.2$.

In our last example we used a shot gather from the Gulf of Mexico (Figure 3a). The decimated shot gather obtained by removing every other trace from the gather is portrayed in Figure 3b. The decimated section is interpolated using local PFs with length equal to 4 and forgetting factor $\lambda = 0.2$. Figure 3c shows the result of the interpolation exercise.

Conclusions

In this paper we introduced a new interpolation method that can handle events that do not conform to the classical assumption of constant dip in the window of analysis. Rather than building an f-x interpolation strategy based on overlapping adjacent windows, we have adopted the Recursive Least Squares method to update the prediction error filters. The local predictor error filters are saved and used to reconstruct the data using the procedure proposed in Spitz (1991) in seminal paper.

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