Structure-and-Amplitude-Preserving Multi-Channel Deconvolution

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Summary

We propose a new noise-reduction method based on multi-channel deconvolution. We pose the noise reduction problem as an inverse problem and propose a cost function that uses *a priori* information about the reflectivity model. In particular, we introduce a smoothing constraint across traces via FX-filtering. Once we have computed the reflectivity model, we predict the data by convolving them with an estimated wavelet. The two-step method yield clean data without destroying amplitude information. Subtle structural information such as residual moveout is also preserved since we adopt FX prediction filtering as a soft preconditioner. A test with gathers from the Marmousi data set shows that the method provides clean and well-focused AVO gathers.

Introduction

Noise-reduction is important in seismic data processing. Noise compromises our ability to depict the earth model. Noise can be classified into coherent noise and random noise according to its predictability. Coherent noise can be predicted using model-based approaches. For example, multiples after NMO exhibit parabolic moveout and therefore the parabolic Radon Transform can be used to model the multiples in the transform domain. The multiples are transformed back to the data space and finally, are subtracted from the original data (Hampson, 1996; Sacchi and Ulrych, 1995; Ng and Perz, 2004). While the latter is an example of a model-based *coherent* noise removal method, it is important to stress that this idea is also applicable to *random* noise attenuation with the important difference that the signal not the coherent noise is modelled. The difference between the data and the modelled signal is an estimate of the incoherent noise. An example of the latter framework are the well known algorithms for noise suppression in the frequency-space domain: FX filtering and FX projection filtering algorithms proposed by Canales (1984) and Soubaras (1994), respectively.

It has been shown that regularized migration/inversion using *a priori* information of the model can yield high quality images of the subsurface (Wang et al., 2005). In this paper, we use a similar idea for noise reduction by solving a multi-channel deconvolution problem. Since seismic events are usually predictable within a gather, we use FX domain prediction filters to constrain the spatial behaviour of the reflectivity model. The goals are two-fold: applying prediction filters helps to reduce

noise, and multi-channel deconvolution helps to honor the input data. The combined application of these two techniques is used to develop a framework for amplitude-preserving noise attenuation.

Theory

We start our analysis with the following simple equation for random noise reduction:

$$d_{obs} = d + e,\qquad\qquad(1)$$

where d_{obs} is the observed data, d is the clean coherent data, and e is the random noise. The denoising problem can be solved by minimizing the following cost function:

$$J(d) = \| d_{obs} - d \|^2 \qquad\qquad(2)$$

which assumes noise with a Gaussian distribution. More straightforwardly, we try to minimize the difference between the predicted and observed data. In addition we represent the clean data as the convolution of wavelet with a reflectivity sequence:

$$d = Lm,\qquad\qquad(3)$$

where L is a convolution matrix which contains the wavelet, and m is the reflectivity model. After combining equation (2) and (3), we obtain

$$J(m) = \| d_{obs} - Lm \|^2.\qquad\qquad(4)$$

Once we found the optimal model $\hat{m}$, we can predict the data by

$$d_{pred} = L\hat{m}.\qquad\qquad(5)$$

Equation (4) can be further developed using preconditioning strategy. For example, an ideal CMP gather should be smooth along the offset axis. Using this *a priori* information we stabilize equation (4) by introducing a model-space constraint in preconditioned mode (Hanke and Hansen, 1993; Prucha and Biondi, 2002; Fomel and Claerbout, 2003) as below:

$$J(z) = \| d_{obs} - LPz \|^2,\qquad\qquad(6)$$

where P is a preconditioner and z is an auxiliary variable. Equation (6) can be generalized to the multi-channel case as follows:

$$J(Z) = \sum_{k=1}^{NTR} \| d_k - L(PZ)_k \|^2,\qquad\qquad(7)$$

where d_k denotes the *k*-th seismic trace in the time domain, Z is the ensemble of reflectivity traces $Z = (z_1, z_2, ..., z_{NTR})$ in the time domain, P is a multi-channel preconditioning operator (prediction filter) which is applied to Z. The variable $(PZ)_k$ denotes the *k*-th cleaned reflectivity trace. The application of the operator P involves the following operations: converting data to the FX domain via the Fourier transform, applying a spatial prediction filter and converting back to the TX domain via the inverse Fourier transform. We minimize the above cost function by the Conjugate Gradient (CG) algorithm (Hestenes and Steifel, 1952) and control the data fitting by the number of CG iterations. The prediction filters, which are required for the operator P, can be extracted from the adjoint solution for each channel (crosscorrelation of the estimated wavelet with the data):

$$\tilde{m}_k = L'd_k, k = 1,2,...,NTR,\qquad\qquad(8)$$

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where $\widetilde{m}_k$ denotes the k-th adjoint solution. We follow Canale's approach (1984) to compute prediction filters for a band of frequencies. The prediction filters are saved or kept in memory since they will become part of our operator when minimizing equation (7).

Equation (7) shows that in the modeling routine we should sequentially apply two operators P and L, i.e. first we clean up the model by applying the prediction filter, and then we multiply the result with the convolution matrix L which contains the wavelet. In the adjoint routine we apply the adjoint (transpose) of the convolution matrix first to the data, then we apply the adjoint of the prediction filtering. The adjoint of equation (7) can be expressed as

$$\widetilde{z} = P'L'd ,\qquad(9)$$

where $\widetilde{z}$ denotes the output of the adjoint operator. The processing flow for our new denoising algorithm can be summarized as follows

1. Estimate the seismic wavelet and compute the convolution matrix L.
2. Compute the adjoint solutions using equation (8) and use them to estimate prediction filters.
3. Minimize cost function (7) via the CG method.
4. Predict the clean data using equation (5).

Synthetic Example

We tested the algorithm with a synthetic CMP gather from the Marmousi data. We added significant noise to data to compare the performance of the conventional FX-filtering and the proposed method. Note that this is a local noise-reduction test and no nearby CMP information was used. Figure 1 shows that the new algorithm tends to provide more coherent and cleaner output than the conventional FX-filtering algorithm. In addition, subtle structural information and the AVO signature are better preserved by the new method.

Conclusions

We have proposed a new multi-channel noise reduction algorithm using TX domain deconvolution and FX domain prediction filtering. The prediction filters are computed once and saved as a preconditioner for a multi-channel inverse problem. Synthetic examples were used to test the performance of the proposed method. We have showed that the new method better preserves the AVO signature than standard FX filtering techniques.

Acknowledgements

We thank Divestco Inc. for allowing us to publish this paper. We also thank colleagues at Divestco Processing for their valuable suggestions.

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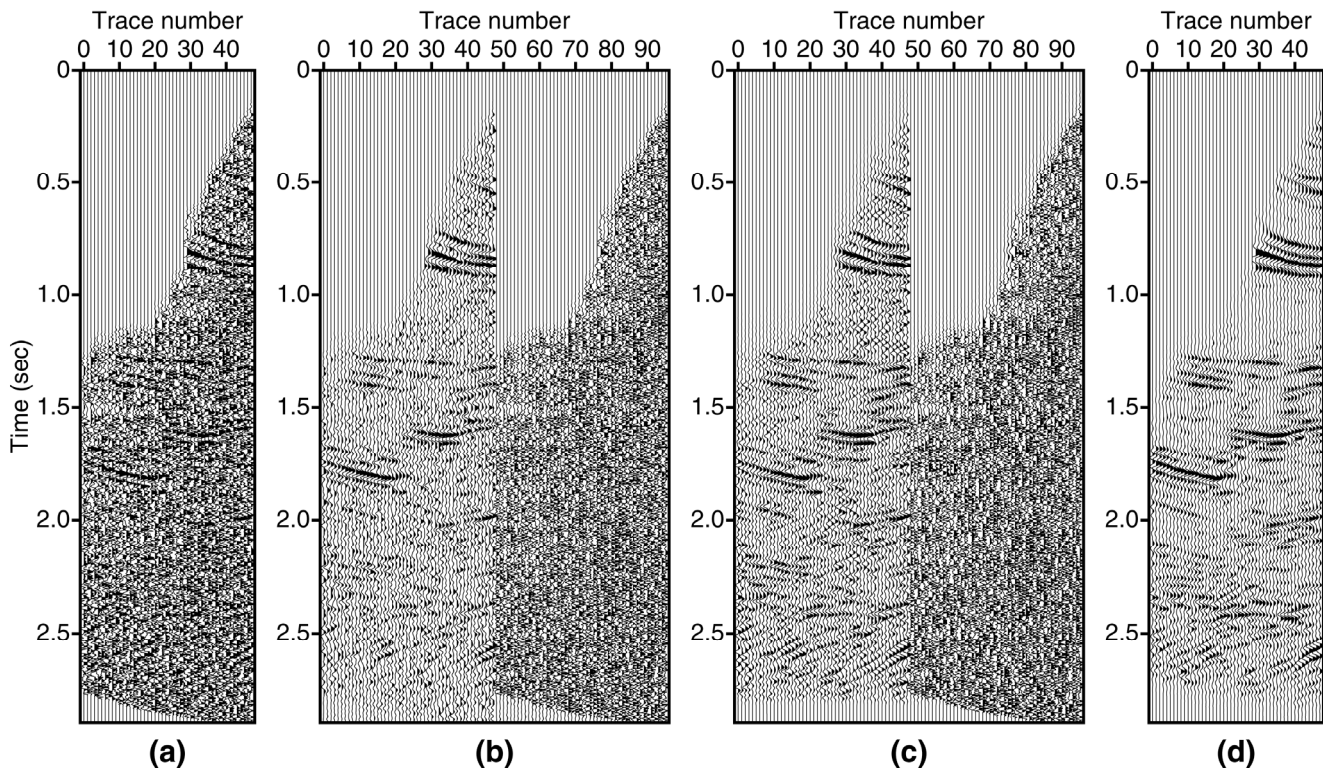
Figure 1: Comparison of two noise-reduction methods using a synthetic dataset (Marmousi data with additive noise). (a) Input CMP gather. (b) Predicted data using Canales' method (1984) and the residuals. (c) Predicted data using the new method and the residuals. (d) Noise-free reference gather.