Curvelet-Regularized Seismic Deconvolution

Vishal Kumar*
University of British Columbia, Vancouver, BC
vkumar@eos.ubc.ca

and

Felix Herrmann
University of British Columbia, Vancouver, BC, Canada

Summary

There is an inherent continuity along reflectors of a seismic image. We use the recently introduced multiscale and multidirectional curvelet transform to exploit this continuity along reflectors for cases in which the assumption of spiky reflectivity may not hold. We show that such type of seismic reflectivity can be represented in the curvelet-domain by a vector whose entries decay rapidly. This curvelet-domain compression of reflectivity opens new perspectives towards solving classical problems in seismic processing including the deconvolution problem. In this paper, we present a formulation that seeks curvelet-domain sparsity for non-spiky reflectivity and we compare our results with those of spiky deconvolution.

Introduction

In this paper, we address the deconvolution problem for which we assume to have a known source wavelet and an unknown reflectivity. The forward problem can be written as:

$$y = Am + n \ , \tag{1}$$

where y is the observed data, A is the convolution operator (Toeplitz matrix), m is the reflectivity and n white zero-centered is Gaussian noise. Given A and y, we need to find m.

Since the early 80's , researchers have cast this problem as a l_1-norm minimization (Oldenburg et al., 1981), where the reflectivity is assumed be made up of spikes. In recent work by Felix Herrmann, it was shown that the assumption of spiky reflectivity is too limited to describe seismic reflectivity (Herrmann and Bernabe, 2004). This means that in cases where the reflectivity is not spiky, spiky deconvolution will fail (Herrmann and Bernabe, 2004; Herrmann, 2005).

In our approach, we exploit continuity along reflectors for non-spiky reflectivity. We show that the non-spiky reflectivity is sparse in the curvelet-domain and that this sparsity can be exploited while solving the deconvolution problem. We start with a brief introduction to curvelets, followed by presentation of our algorithm and application on synthetic data.

Curvelets

Curvelets are amongst one of the latest members of the family of multi-scale and multi-directional transforms (Candes and Donoho, 1999; Candes and Donoho, 2002). They are tight frames with moderate redundancy. Different curvelets at different frequency are shown in Figure-1. A curvelet is strictly localized in frequency and pseudo-localized in space (has a rapid decay). In the physical domain, the curvelets are oscillatory in one direction and smooth in other. The construction of curvelets is such that any object with C^2 singularities can be represented by very few curvelet coefficients (Candes and Donoho, 2002). The sparsity of curvelets for C^2 singularities makes it an ideal choice for estimating the reflectivity.

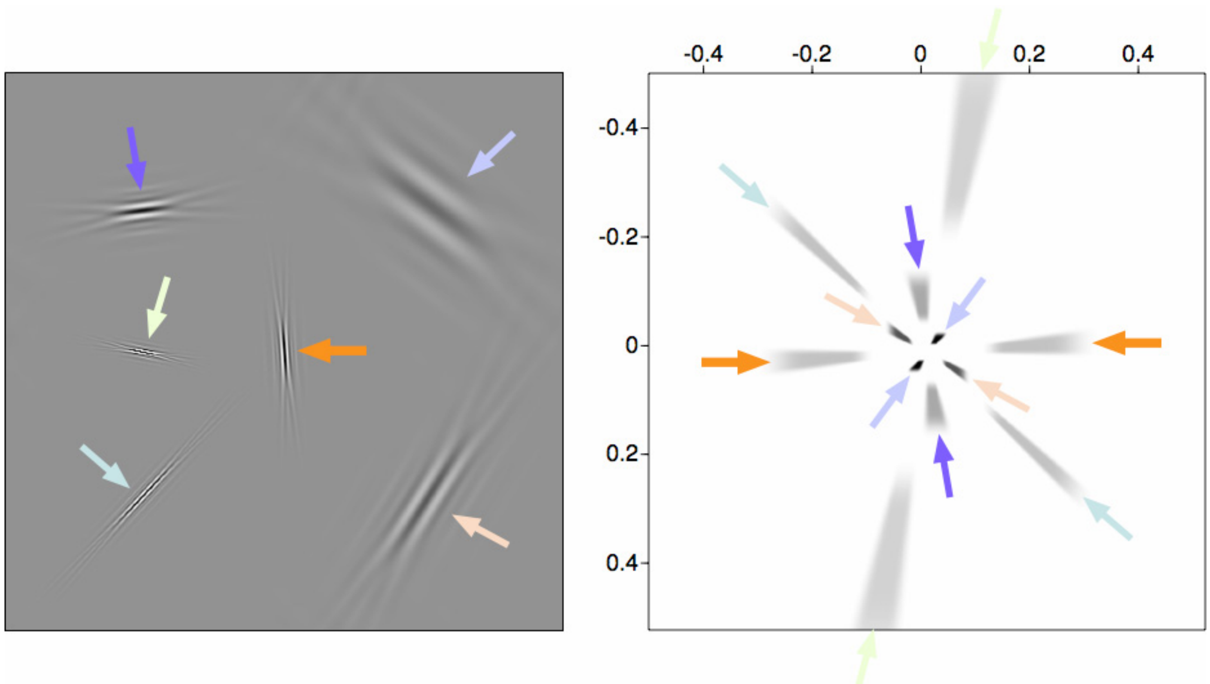

Figure 1: A few curvelets in both spatial (left) and frequency domain (right) (adapted from Herrmann and Hennenfent, 2007)

Method

The deconvolution problem can be cast as following constrained optimization problem:

$$\begin{cases} \min_{x} \|\mathrm{x}\|_1 \text{ subject to } \|\mathrm{y} - \mathrm{AC}^{\mathrm{T}}\mathrm{x}\|_2 \leq \varepsilon \\ \tilde{\mathrm{m}} = \mathrm{C}^{\mathrm{T}}\tilde{\mathrm{x}} \ , \end{cases} \tag{2}$$

where $\tilde{\mathrm{x}}$ is the curvelet coefficient vector, ε is proportional to the noise level, C^T is the curvelet synthesis operator and $\tilde{m}$ is the estimated reflectivity. The above constrained optimization problem is solved by a series of following unconstrained optimization problem (Hennenfent et al., 2005) :

$$\tilde{\mathrm{x}} = \arg\min_{x} \frac{1}{2}\|y - \mathrm{AC}^{\mathrm{T}}\mathrm{x}\|_2^2 + \lambda\|x\|_1 \ , \tag{3}$$

where λ is the trade-off parameter. We solve a series of above optimization problem (3) starting with a high λ and decreasing the value of λ until $\|\mathrm{y} - \mathrm{AC}^{\mathrm{T}}\mathrm{x}\|_2 \approx \varepsilon$, which corresponds to the solution of our optimization problem. By solving (3) we try to find the sparsest set of curvelet coefficients which explains the data within the noise level (Hennenfent et al., 2005). The lowering of λ is done in a controlled way so that we reach the optimum λ very fast using the SPG l_1 algorithm (Van den Berg and Friedlander, 2008; Hennenfent, 2007). Details on the algorithm can be found in the SPG l_1 Technical report (Van den Berg and Friedlander, 2008).

Results

Jon Claerbout's Sigmoid model is one and half times fractionally diffrentiated in the frequency domain to obtain a non-spiky reflectivity model. Notice that the reflectivity is no longer made up of spikes. The noisy data is obtained by convolving a Ricker wavelet with the synthetic reflectivity and random Gaussian noise was added. We then apply our algortihm to estimate the reflectivity, assuming we know the source wavelet. For comparison, we also do spiky deconvolution on the same data. Figure-2 shows data, original reflectivity and estimated reflectivity with curvelets and spiky deconvolution. Figure-3 shows zoom-in on a single trace for original and estimated reflectivity with curvelets and spiky deconvolution. In case of spiky deconvolution, the algorithm tries to find a series of spikes which explains the data but in this case the reflectivity is no longer made of spikes and thus spiky deconvolution fails. On the other hand, our curvelet-regularized deconvolution algorithm which exploits the continuity along reflectors yields better results.

Conclusions

In this paper, the two dimensional structure of reflectivity is exploited by curvelets. We showed how non-spiky reflectivity can be recovered by exploiting the continuity along the reflectors by promoting curvelet-domain sparsity. The assumption of spiky reflectivity is too limited may not be valid in all cases. Thus sparsity of such a type of reflectivity in the curvelet-domain is a strong prior which we can use as part of our deconvolution algorithm.

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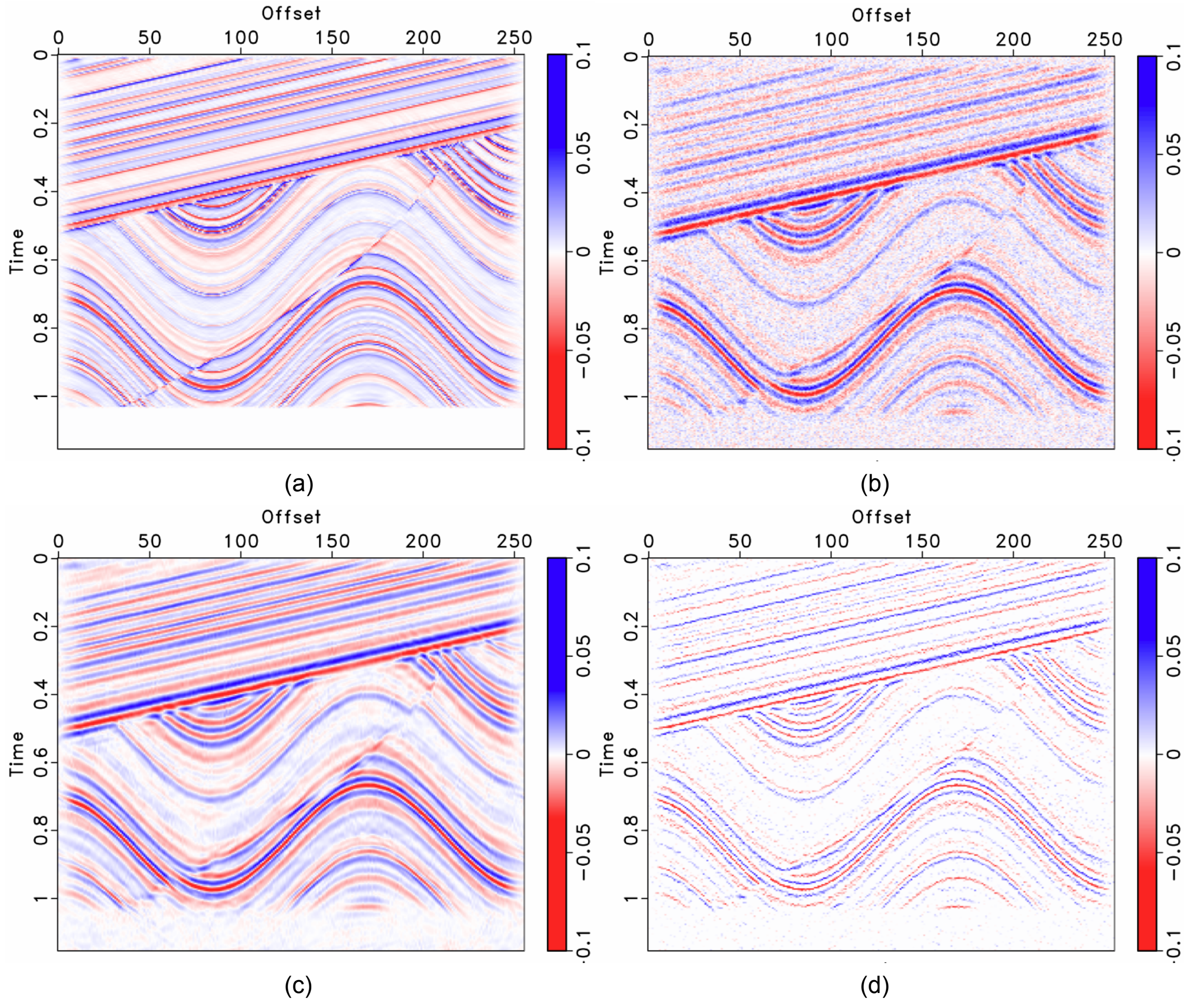

Figure 2: Synthetic data and reflectivity estimates. (a) Original non-spiky reflectivity. (b) Noisy data (SNR~7db). (c) Estimated reflectivity with curvelets. (d) Estimated reflectivity with spiky deconvolution.

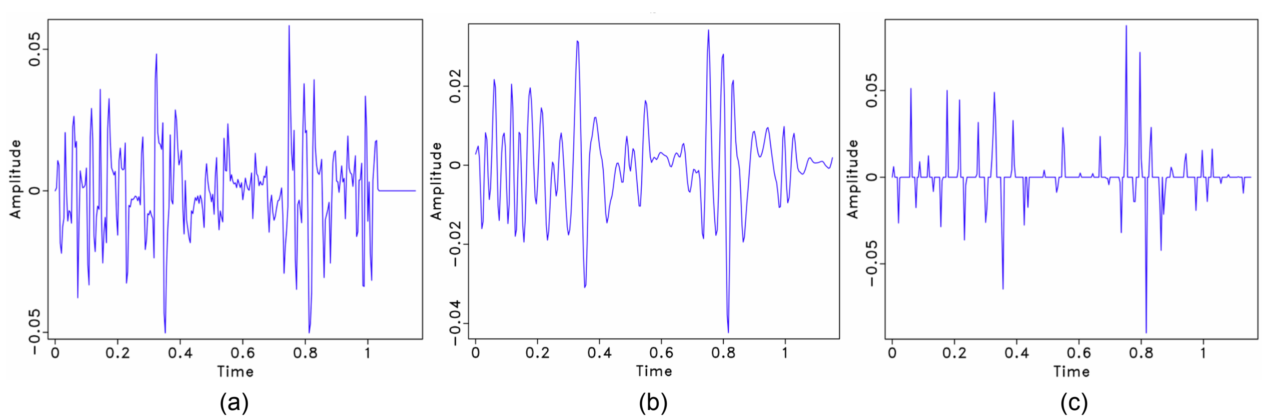

Figure 3: One trace plot of reflectivity. (a) Original reflectivity. (c) Estimated reflectivity with curvelets. (d) Estimated reflectivity with spiky deconvolution.