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Resolvability Analysis of Single Azimuth Seismic Moment Tensor Inversion

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Summary

Microseismicity monitoring of hydraulic fractures is frequently done with a single azimuth observational geometry with receivers deployed in a vertical array. The latter restricts the solutions that we are capable to retrieve and can hide the existence of non-deviatoric sources (Vavrycuk, 2007). The complete recovering of the source mechanisms is an important aspect for the understanding of induced fracturing and can add key information for the characterization of the fracturing process (i.e., whether the fracture is created as a propagating tensile opening or via multiple shearing fractures). In this presentation we analyze the resolution matrix of the Seismic Moment Tensor (SMT) inversion algorithm and analyze the resolvability problem as a function of azimuth for different type of mechanisms.

Introduction

SMT inversion is routinely used in global seismology (Dziewonski et al., 1981; Zhao and Helmberger, 1994). Nowadays, the capability to solve for the full SMT is achieved due to a large number of available surface stations and a broad azimuthal coverage. Other applications where the SMT inversion has probed to be a valuable source of information are mining activity monitoring (Trifu and Shumila, 2002; Feignier and Young, 1992) and hydraulic fracture monitoring (Nolen-Hoeksema and Ruff, 2001; Bleakly et al., 2007). We present a SMT inversion algorithm (Nolen-Hoeksema and Ruff, 2001) and via SVD resolution analysis we analyze the resolvability of the SMT solution as a function of azimuth.

Theory

We start with the equations relating the far-field displacement and the SMT rate in a homogeneous elastic space (Aki and Richards, 1980, Nolen-Hoeksema and Ruff, 2001)

$$U_i^P(\mathbf{x},t) = \frac{1}{4\pi\rho\alpha^3 r} R^P \dot{M}\left(t - \frac{r}{\alpha}\right) \quad (1) \qquad U_i^S(\mathbf{x},t) = \frac{1}{4\pi\rho\beta^3 r} R^S \dot{M}\left(t - \frac{r}{\beta}\right) \quad (2)$$

where U_i is the displacement observed for P or S waves in the i-th component of a receiver, r is the distance between the source and the receiver, $\mathbf{x}$ is the receiver position in a Cartesian reference system, ρ is the density of the medium, α and β are seismic velocities for P and S waves, respectively. The variable $\dot{M}$

indicates the SMT rate function, and R^P, R^S are the radiation pattern coefficients for P and S waves. Integrating in time both sides of equations (1) and (2) leads to the following expressions

$$\int U_i^P(\mathbf{x},t)dt = \frac{1}{4\pi\rho\alpha^3 r}\mathbf{r}^P\mathbf{m} \qquad (3) \qquad\qquad \int U_i^S(\mathbf{x},t)dt = \frac{1}{4\pi\rho\beta^3 r}\mathbf{r}^S\mathbf{m} \qquad (4)$$

In equations (3) and (4) $\mathbf{r^P}$, $\mathbf{r^S}$ are row vectors containing the radiation pattern coefficients for P and S waves, **m** is a column vector containing the 6 independent elements of the SMT in the order M_{11}, M_{12}, M_{13}, M_{22}, M_{23} and M_{33}. For our work we considered a homogeneous half-space with $\alpha = 2500$ m/s, $\beta = 1440$ m/s and $\rho = 2.5$ g/cm^3. Expressions (3) and (4) can be organized in matrix form as follows

$$\mathbf{G\ m = d} \qquad (5)$$

In this expression the matrix **G** contains information about the properties of the medium and the geometry of the problem. The vector **d** contains the integrated observed displacements for the corresponding component, receiver and wave type. We carried on the inversion by evaluating the least squares solution to equation (5)

$$\mathbf{m} = (\mathbf{G}^T\mathbf{G})^{-1}\,\mathbf{G}^T\mathbf{d} \qquad (6)$$

The resolution matrix is defined as (Van Rijssen and Herman, 1991)

$$\mathbf{R} = \mathbf{G}^{\dagger}\mathbf{G} \qquad (7)$$

where $\mathbf{G}^{\dagger}$ is the pseudo inverse of the matrix **G** computed with the singular value decomposition (SVD) method. From (7), it can be seen that when $\mathbf{G}^{\dagger} = \mathbf{G}^{-1}$ the resolution matrix is equal to the identity. In this case all the elements of the model are linearly independent and can be unambiguously retrieved. In reality, the matrix is not an identity. Non-zero off-diagonal values in **R** mean that the corresponding diagonal element is linearly related to off-diagonal components (cross-talk). In this situation the dependent elements become more sensitive to noise and are solved with inaccuracy (Jing and Rape, 2004).

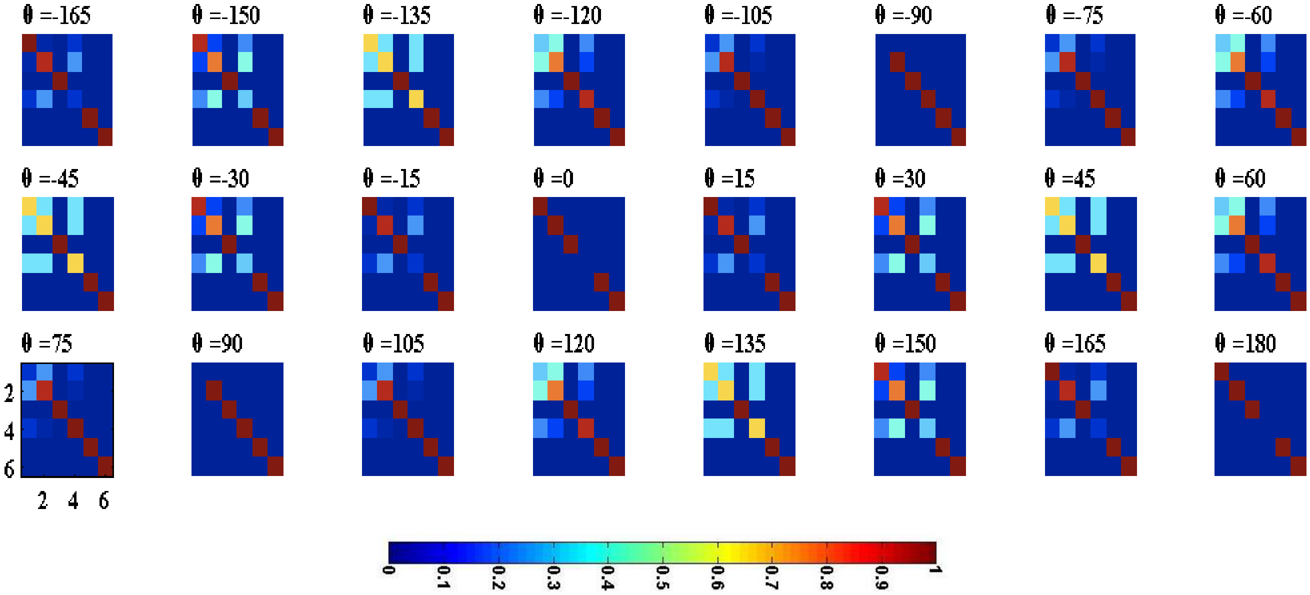

Figure 1: Resolution matrices for each observational azimuth of a vertical line of 10 receivers distributed symmetrically in the vertical direction from the source level. θ is the azimuth from North that defines the plane formed by the source position and the line of receivers.

Resolution analysis

We set a vertical line of 10 receivers at 400 m from the source position, the receivers are separated every 50 m and are symmetrically distributed above and below the source level in the vertical direction. We moved the line of receivers around the source at steps of 15° and for each azimuth we computed the corresponding resolution matrix (see Figure 1). Each element of the diagonal of the matrices represents an element of the SMT arranged in the order $R_{11} = M_{11}$, $R_{22} = M_{12}$, $R_{33} = M_{13}$, $R_{44} = M_{22}$, $R_{55} = M_{23}$, and $R_{66} = M_{33}$. The off-diagonal boxes represent the cross-talk between the elements that intercept in the diagonal for that box. Diagonal elements with values close to 1 can be completely solved by the inversion, as opposite to elements close to 0. As expected, when the observational azimuth of our line of receivers is aligned with the reference system, 5 out of 6 elements of the SMT are linearly independent and can be retrieved via inversion (Vavrycuk, 2007). These results suggest that the resolvability of the inversion depends primarily on the orientation of the reference system that we choose to solve the problem.

When our reference system is aligned with our observational azimuth, we are able to retrieve the maximum quantity of information from our observed traces (5 independent elements of the SMT). On the other hand, as our observational azimuth moves away from the main axis of our reference system, cross-talk exists between the elements M_{11}, M_{12} and M_{22} and then, the inversion becomes more unstable and the results for these 3 elements are inaccurate. Imposing the constraint over the trace of the SMT could alleviate the instability but the resulting inverted components are only reliable if the source mechanism honors the constraint (Vavrycuk, 2007). An example of the inversion results in two different reference systems is showed in Table 1. Three source mechanisms were tested. In these cases, the trace of the tensor was constrained to be zero. In other words, only deviatoric solutions that fit the observations were considered. Notice that, for the isotropic mechanism, the inversion retrieves 3 inaccurate components of the SMT when the reference system is not aligned with the observational plane, but 5 reliable elements are found for the aligned case. For deviatoric sources, the 6 elements of the SMT are reliable in the inversion on both reference systems.

Table 1. Synthetic tests with noise-free data. Results are showed for 3 different mechanisms, a double couple, an explosion and a Compensated Linear Vector Dipole (CLVD). The inversion was run with the trace constrained for the deviatoric SMT in all cases, in the first set of results the observational plane is aligned with the reference system, in the other set, the plane is at 45°.

Mechanism 1: Double Couple $\phi = 60°, \delta = 50°, \lambda = 60°$				Mechanism 2: Explosion			Mechanism 3: CLVD		
M	M true	M inverted		M true	M inverted		M true	M inverted	
		Azimuth Aligned	Azimuth at 45°		Azimuth Aligned	Azimuth at 45°		Azimuth Aligned	Azimuth at 45°
M_{11}	-0.9714	-0.9714	-0.9714	1	1.0000	-0.5000	1	1.0000	1.0000
M_{12}	0.1778	0.1778	0.1778	0	0.0000	1.5000	0	0.0000	0.0000
M_{13}	-0.0305	-0.0305	-0.0305	0	0.0000	0.0000	0	0.0000	0.0000
M_{22}	0.1185	0.1185	0.1185	1	-2.0000	-0.5000	-2	-2.0000	-2.0000
M_{23}	-0.3535	-0.3535	-0.3535	0	0.0000	0.0000	0	0.0000	0.0000
M_{33}	0.8529	0.8529	0.8529	1	1.0000	1.0000	1	1.0000	1.0000

Conclusions

The information displayed in the resolution matrices is consistent with previous work for the cases when the reference system is aligned with the observational azimuth of our receivers. When the reference system is not aligned with the observational plane, the elements M_{11}, M_{12} and M_{22} of the SMT display cross-talk and introduce instability into the inversion process. Further work entails also the study of acquisition design strategies that maximize the resolvability of the SMT.

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