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# Curvelet Denoising: Application to Crustal Reflection Data

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## Summary

The separation of signal and noise is a key issue in seismic data processing. By noise, we refer to the incoherent noise that is present in the data. We make use of the parsimonious representation of seismic data in the curvelet domain to perform the noise suppression. The problem can be cast as an inverse one and the solution is found by updating results at each iteration. In this work, we extend the process of denoising with curvelets to deep crustal seismic data. After standard processing, we applied the iterative curvelet denoising to the post-stack data. Comparing the results with those from F-X deconvolution, curvelets clearly perform better than the latter by attenuating the noise with minimal harm to the signal.

## Introduction

Incoherent noise present in seismic reflection data corrupts the quality of the signal and can often lead to misinterpretation. Thus, separation of signal and noise is an important issue in seismic data processing, particularly in crustal data where the signal-to-noise ratio is low. The forward problem can be written as:

$$y = m + n \tag{1}$$

where $y$ is the known noisy data vector, $m$ is the unknown noiseless data vector and $n$ is zero-centered white Gaussian noise. Our objective is to recover $m$.

In seismic data, recorded wave fronts (i.e., reflections) arise from the interaction of the incident wave field with inhomogeneities in the Earth's subsurface and correspond to so-called two-dimensional singularities. The wave fronts can become contaminated with noise during acquisition or even due to processing problems. In the curvelet domain, the signal is sparse whereas noise is not. In other words, signal and noise have minimal overlap in the curvelet domain (Neelamani et al., 2008). This makes the curvelet transform an ideal choice for detecting wave fronts and suppressing noise. We apply curvelet denoising to post-stack crustal seismic reflection data recorded along LITHOPROBE's SNorCLE Line 1 situated in the Northwest Territories (Cook et al., 1999). The denoised seismic profile shows much more coherent reflectivity.

## Theory and Method

As seismic data are contaminated with random noise, many methods have been developed to suppress such incoherent noise. Some of these techniques discriminate between signal and noise based on their frequency content (bandpass filter) or they use some sort of prediction filter (F-X deconvolution). Such methods do remove the random noise but at the same time they may remove some of the signal (Neelamani et al., 2008). More recently, wavelet processing has been applied for noise suppression. The wavelet transform is good for point-like events (one-dimensional singularities). However, for higher dimension singularities, wavelets fail to give a parsimonious representation of an image.

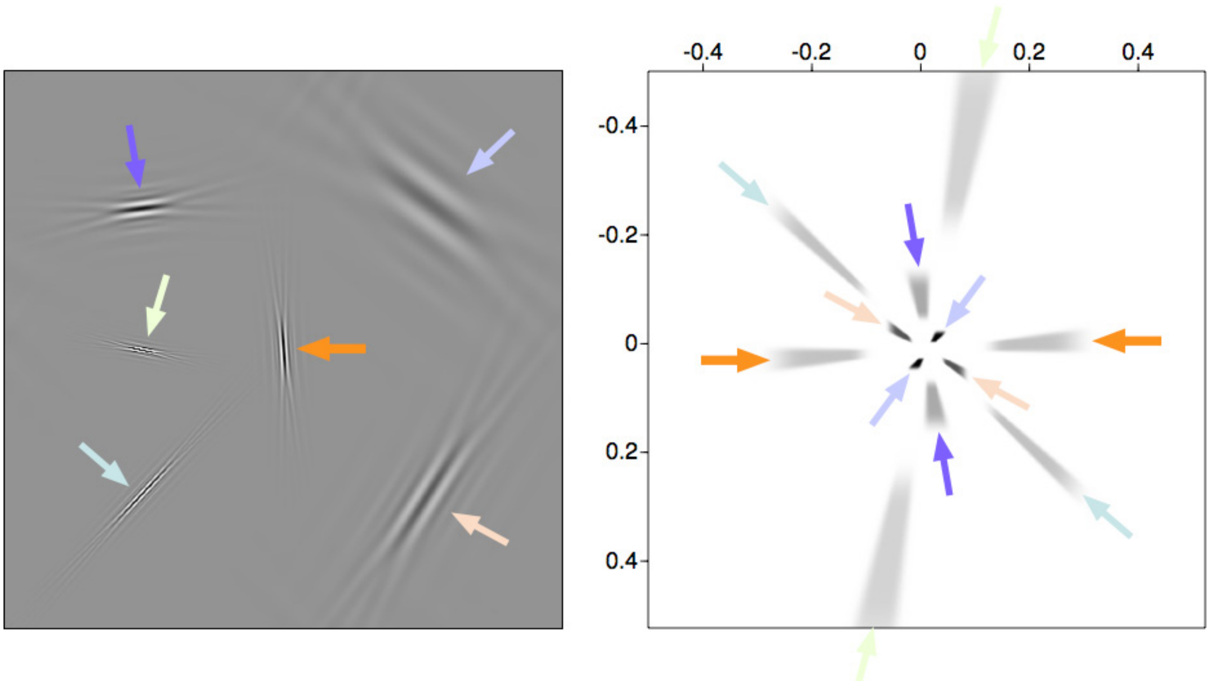


Figure 1: A few curvelets in both spatial (left) and frequency domain (right); adapted from Herrmann and Hennenfent (2007). The colored arrows identify the same curvelets in the two domains.

Curvelets are amongst one of the latest members of the family of multiscale and multidirectional transforms (Candes et al., 2006). A curvelet is strictly localized in frequency and pseudo-localized in space; i.e., it has a rapid spatial decay. In the physical domain, curvelets look like little plane waves that are oscillatory in one direction and smooth in perpendicular directions. Each curvelet is associated with a position, frequency band-width and a dip. Different curvelets at different frequencies, angles and positions are shown in Fig. 1. The construction of curvelets is such that any object with a wavefront-like structure (e.g., seismic images) can be represented by a relatively few significant transform coefficients (Candes et al., 2006).

The denoising problem with curvelet transform-domain sparsity can be cast into the following constrained optimization problem:

$$\begin{cases} \min\limits_{x} \|\mathrm{x}\|_1 \text{ subject to } \|\mathrm{y} - \mathrm{S}^{\mathrm{H}}\mathrm{x}\|_2 \leq \varepsilon \\ \tilde{\mathrm{m}} = \mathrm{S}^{\mathrm{H}}\tilde{\mathrm{x}} \ , \end{cases} \tag{2}$$

where $\tilde{\mathrm{x}}$ represents the estimated transform coefficient vector, $S^H$ is the transform synthesis operator and $\varepsilon$ is proportional to the noise level. By solving Eq. 2, we try to find the sparsest set of transform coefficients which explains the data within the noise level (Hennenfent et al., 2005). The final estimated model is given by $\tilde{m}$. In our case, the above constrained optimization problem (Eq. 2) is solved by a series of the following unconstrained optimization problems (Herrmann and Hennenfent, 2008):

$$\tilde{x} = \arg\min_{x} \frac{1}{2}\left\| y - S^{H}x \right\|_{2}^{2} + \lambda \left\| x \right\|_{1} \tag{3}$$

where $\lambda$ is the regularization parameter that determines the trade-off between data consistency and the sparsity. We solve a series of such problems (Eq. 3) starting with high $\lambda$ and decreasing its value until $\left\| y - S^{H}x \right\|_{2} \approx \varepsilon$, which corresponds to the solution of our optimization problem (Lustig et al., 2007). For this work, S represents the curvelet transform. By solving (3) we try to find the sparsest set of curvelet coefficients that explain the data within the noise level (Hennenfent et al., 2005).

## Results

Along SNorCLE Line 1, the clarity of the deep crustal seismic data is degraded by the low S/N ratio, which precludes detailed interpretation of the reflectivity images. The stacked seismic profile, a 60-km-long segment of which is shown in Fig. 2a, shows a decrease in reflectivity between the crust and upper-mantle; the crust-mantle transition is identified at ~11.0 s (~34 km depth). On the southwestern end of the segment shown, the crust-mantle transition becomes more diffuse due to high background noise. The reflection data were processed following standard procedures used in LITHOPROBE studies (e.g., Cook et al., 1999). Curvelet denoising was applied and tested on the post-stack data as an alternative to F-X deconvolution.

The improvement in image resolution is clear when comparing the two data sets before and after the curvelet denoising (Figs. 2a and 2b, respectively). The denoised seismic image shows a highly reflective crust, a transparent upper mantle and a relatively flat Moho but with an offset of about 1 s at about CDP 1100. The lower crust contains a series of discrete, east-dipping reflections within a zone that is 2.0 to 3.0 s thick (about 6.5 to 10 km) and flattens into the Moho. Below about 9.5 s at the southwestern end of the section, the reflectivity becomes sub-horizontal to about the position of the Moho offset. These events were not visible on the original stacked image. The Moho is characterized by a sharp, narrow band of reflections (200-300 ms) that are piece-wise continuous, except for the offset, over the 60-km-long segment. The difference plot between the original data and the curvelet result (Fig. 2c) shows that the noise removed by curvelet processing is random in nature. Thus, the technique achieves maximum noise attenuation without removing coherent events. The F-X deconvolved image (Fig. 2d) shows most of the features observed on the curvelet denoised image but the NE-dipping and sub-horizontal reflectivity is not as clearly expressed. We conclude that the results from curvelet processing are superior to those from F-X deconvolution.

We also have applied the algorithm to single-shot gathers and identified reflections from the Moho where none could be observed prior to denoising. The next phase of our research will be to undertake careful depth and angular weighting to further enhance resolution of the seismic images. Using 3-d curvelets, the algorithm can be extended to apply similar denoising procedures to 3-d data sets.

## Conclusions

We demonstrate that incoherent noise in deep crustal reflection data can be effectively suppressed using curvelet processing procedures and that the results are better than those from more conventional methods, including F-X deconvolution. The algorithm is able to remove incoherent noise and improve the resolution of post-stack data without affecting the coherent reflectivity. The sparsity of wavefront-based seismic images in the curvelet-domain is a primary property of which our curvelet frame denoising algorithm takes advantage.

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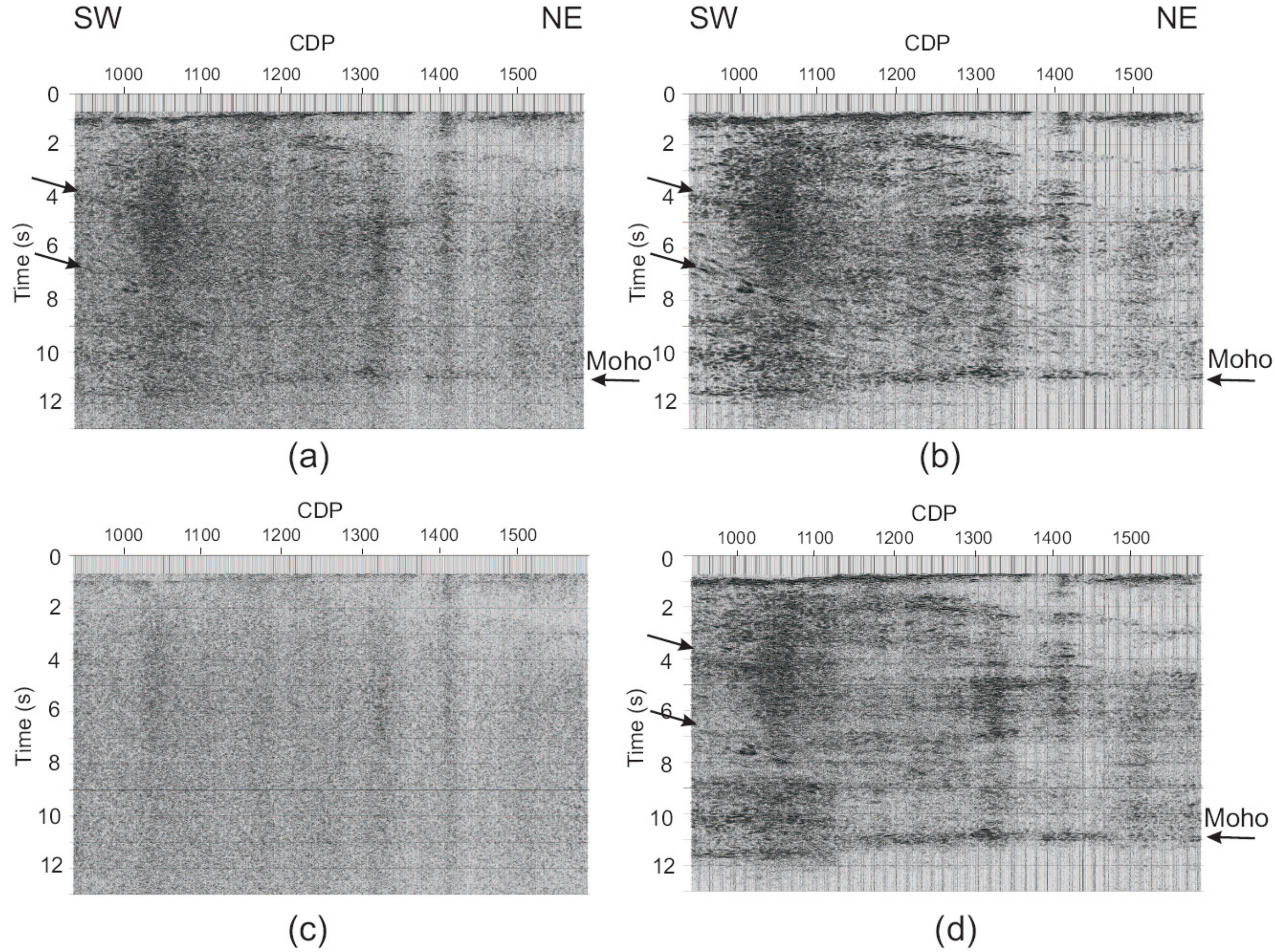


Figure 2: (a) The original stacked seismic section, ~60 km long, contaminated with random noise. Arrows indicate some of the features discussed in the text. (b) Result of applying the curvelet denoising algorithm to the same section. (c) Difference plot between (b) and (a). (d) Result of applying F-X deconvolution to the same section.

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