Stabilizing Seismic Absorption Compensation

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Summary

To overcome the instability problem of inverse Q filtering, the introduced method formulates seismic absorption compensation (a.k.a. De-absorption) as an inverse problem based on the statistical theory. The matrix of forward modeling is composed of the time-variant wavelets. The de-absorption is solved by an iterative conjugate gradient approach. This scheme is tested on both synthetic and real data. The results of de-absorption are related to the accuracy of the estimated Q values and also of the seismic wavelets.

Introduction

Seismic absorption compensation is commonly implemented by means of inverse Q filtering or spectrum whitening (Yilmaz, 2000). Inverse Q filtering is inherently unstable since the inverse operator will boost high frequency noise. To ensure that noise is not unnecessarily amplified, it is important to design the inverse operator appropriately at high frequencies. One way to avoid the instability is to use a band-limited version of the inverse operator, i.e., to replace the amplitude compensation operator by its low-pass version, which is basically partial compensation. In order to stabilize the process of absorption compensation, the inverse Q filtering problem is formulated as an inverse problem using a Cauchy-Gauss model, and solves the problem using conjugate gradient method.

Method

Inverse problems can be approached from the point of view of probability theory (Tarantola, 1987, Ulrych and Sacchi, 2006). In the statistical theory, it is common to consider the measured data d as uncertain. That is, although there exist true data, we do not know them. The measured data can then be considered as random variables whose mean is that of the true value. Traditionally, an assumption is made that the observed data are random with a Gaussian distribution. This leads to the well known χ^2 test for goodness of fit and to a L_2-norm solution. The solution to the inverse problem is the model vector m. It is not unique, but with some constraints we can obtain a reasonable estimate of m. Each of the model elements can be viewed as a random variable, and thus each estimate of the model is just a realization of a random process.

Statistical inversion theory is, commonly, based on Bayes's Theorem. Because in an inverse problem, we always have observed data d, $p(m|d)$ is a posterior probability. A common approach in Bayesian inversion aims at maximizing $p(m \mid d)$. After data are observed, $p(d \mid m)$ is named the likelihood and it is a measure of the possibility of model m that created d. An approach to estimate m from d, in the situation when no additional information about m is available, is to maximize the likelihood $p(m \mid d)$, giving rise to the MAP (maximum a posteriori) solution. By assigning a prior probability distribution $p(m)$, the conditional distribution function $p(d \mid m)$, and maximizing $p(m \mid d)$, an objective function for the inverse problem can be constructed based on Bayesian theory.

The de-absorption problem can be solved using statistical inversion theory which is based on Bayes' theorem. We will consider data contaminated by noise that is normally distributed as $N(0,\sigma_n^2)$, where n represents the noise vector. The conditional distribution of the data is given by

$$p(m \mid d,\sigma_n) = \left(\frac{1}{2\pi\sigma_n^2}\right)^{(M-1)/2} e^{-\left(1/2\sigma_n^2\right)(d-Gm)^2}, \tag{1}$$

where M is the length of the data vector. For a linear system $d = Gm$, here G is called the coefficient or kernel matrix.

If we assume that a seismic wavelet is available; it is either obtained from check-shot survey, or extracted from a seismic trace. It is further assumed that the model parameter (reflectivity m) is sparse. Assuming that all reflectivity function m_k are associated with the standard deviation σ_m , the joint probability density function of all m_k is $p(m \mid \sigma_m) = \prod p(m_k \mid \sigma_m)$. If $p(m_k \mid \sigma_m)$ satisfies a Cauchy distribution of equation.

$$p(m \mid \sigma_m) = \prod\left(1+\frac{m_k^2}{2\sigma_m^2}\right). \tag{2}$$

We have $\ln\left[p(m \mid d,\sigma_m,\sigma_n)\right] = -c(m) - \frac{1}{2\sigma_n^2}(d-Gm)^T(d-Gm)$ where $c(m)$ is a constraint imposed by the Cauchy distribution: $c(m) = \sum \ln\left(1+\frac{m_k^2}{2\sigma_m^2}\right)$ which is a measure of the sparseness of the model. Furthermore, denoting $\varphi_{cg}(m) = -\ln\left[p(m \mid d,\sigma_m,\sigma_n)\right]$, we can observe that maximizing $p(m \mid d,\sigma_m,\sigma_n)$ is equivalent to minimizing φ_{cg} . Therefore, the cost function for the Cauchy-Gauss model is

$$\varphi_{cg} = c(m) + \frac{1}{2\sigma_n^2}(d-Gm)^T(d-Gm) \tag{3}$$

where m is the reflectivity, d is recorded seismic signal in the time domain, G is composed of time-variant wavelets $b_\tau(t-\tau)$. Both t and τ are in the range of a trace length an

$$G = \left[b_0(t-0), b_1(t-1), \cdots, b_M(t-M)\right]^T \tag{4}$$

In this way, one-dimensional absorption compensation is formulated as an inverse problem. The model which leads to the minimum of the cost function is the reflectivity function we want to find. This Cauchy-Gauss model has also been used in acoustic impedance inversion, signal interpolation and extrapolation (Sacchi, et al., 1998).

Examples

The inverse problem of equation (3) can be solved using conjugate gradient (CG) method. The solution of this inverse scheme converges after several iterations. The details of using CG to solve the optimization problem are omitted here.

A processing flow I suggest to do absorption compensation on a stack seismic section is as the following:

1. Extract Q profile by using windowed time-variant spectral analysis (Zhang, 2002),
2. Do pure phase correction,
3. Extract a zero phase seismic wavelet from the auto-correlation of the shallow part of a trace,
4. Solve the inverse problem for reflectivity iteratively using CG method.

An experiment I did on a real stacked seismic section is shown in Figure 1. Figure 1a is the input; the absorption compensated section is shown in Figure 1b. A number of reflections which are originally un-differentiable around time $t = 920 \quad ms$ can be identified after de-absorption. Their lateral continuities have been improved.

Conclusions

The inverse approach to de-absorption that is introduced here differs quite radically from the deconvolution techniques in customary use. The main difference is that the inverse filter is designed using a Bayesian inference approach and is robust with respect to additive noise. The technique described here has very general application. Specifically, since robust Q compensation provides more accurate information concerning both the amplitude and location of the earth's reflectivity, hydrocarbon reservoir characterization is one obvious target for the introduced method.

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References

Sacchi, M., T. J. Ulrych, and C. Walker, 1998, Interpolation and extrapolation using a high resolution discrete Fourier transform: IEEE Transactions on Signal Processing, 46, 31–38.

Tarantola, A., 1987, Inverse problem theory. Methods for data fitting and model parameter estimation: Elsevier Science Publ. Co.

Yilmaz O., 2000, Seismic Data Analysis, Society of Exploration Geophysicist publication.

Ulrych, T. J. and M. D. Sacchi, 2005, Information-based inversion and processing with applications: Elsevier Publishing Company.

Zhang, C . and Ulrych, T.J. , 2002, Estimation of quality factors – an analytical approach, expanded abstract , 2002, CSEG conference.

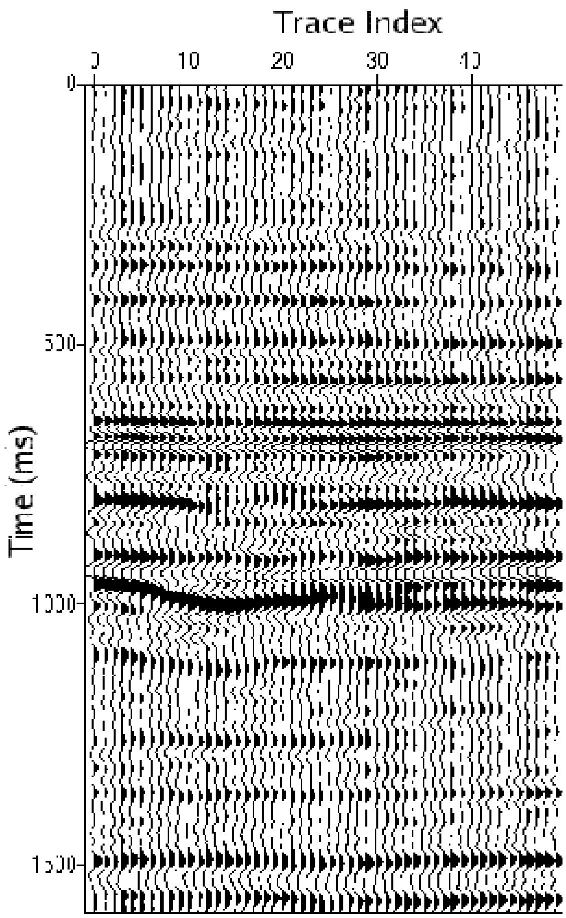

Figure 1a: Input seismic section.

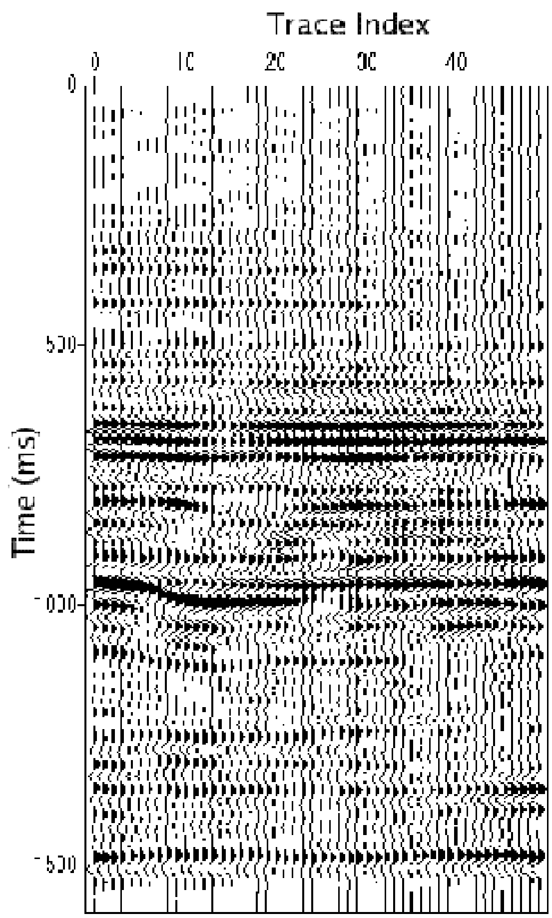

Figure 1b: After absorption compensation.