



Five Dimensional Seismic Data Interpolation

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Summary

Many seismic processing techniques, for example migration, have strict requirements on the regular spatial distribution of traces. Datasets that do not fulfill these requirements, such as most 3D land surveys, will suffer from poor processing results. Although not a substitute for well-sampled field data, interpolation can provide useful data preconditioning that allows these processing techniques to work better.

Interpolation algorithms that use multiple spatial dimensions have many advantages over one-dimensional methods. In particular, simultaneous interpolation in all five seismic data dimensions has the greatest chance to predict missing data with correct amplitude and phase variations. The negative aspect of working in five dimensions is the difficulty of solving the problem in a numerically efficient fashion. In this paper, we discuss an approach that uses Fourier reconstruction in the inline-crossline-offset-azimuth-frequency domain, with a sparseness constraint on the 5D spectrum. The method has been successful for interpolation and regularization of land data in a variety of scenarios, helping on the processing of data with different acquisition problems.

Introduction

The minimum space that completely represents seismic data has five dimensions. Most multidimensional interpolation methods in seismic processing split the data in some kind of single-fold three-dimensional arrangement (stacks, constant offset single azimuth blocks, crosspreads, etc), and infill what is missing on this volume. The reason for this methodology is that most signal processing algorithms used for interpolation work efficiently in three dimensions or less (e.g., Fourier and Radon reconstruction, predictive filtering, etc). However, interpolation of sparse wide azimuth data is more likely to succeed in a full five-dimensional space because amplitude variations are smoother than they are in any projection into a lower dimensional space. As an analogy, consider the shadow of an airplane that flies on a straight line over a mountain range. The shadow of the airplane moves in a very complex path even if the airplane goes in a straight line. Interpolation of the airplane trajectory is much harder on the 2D surface (shadow) than in the original 3D space.

The improvement of interpolation by working on the full 5D seismic space (Trad, 2007) has been confirmed by a large number of case histories. This has been made possible by using a Fourier reconstruction algorithm that can efficiently handle five dimensions simultaneously (see Liu and Sacchi, 2004). The method consists of solving a large inverse problem where the unknown (the “model”) is the desired well sampled seismic data set. A sampling operator maps the model to the acquired data. The five dimensional Fourier spectra of the acquired data along offset, azimuth, inline, crossline and time act as a constraint to define the nature of the missing traces. Among all possible models that are consistent with the input data, this method chooses the

model with the minimum weighted norm 5D spectrum. This constraint tends to ensure that the amplitude changes slowly in all four spatial dimensions while honoring the recorded data.

Because of the multidimensional nature of the process, information from different dimensions can be used simultaneously to infill missing data. This feature permits the algorithm to exploit an important property of 3D survey design, i.e. redundancy of spatial sampling (4 spatial dimensions for a 2D surface) with poor sampling along each dimension (when the other three are fixed). In summary, multidimensional interpolation has the capability to capture amplitude variations along all the dimensions simultaneously and create new data consistent with that information.

Five Dimensional Interpolation

To interpolate data in five dimensions an algorithm is required that can efficiently handle large volumes of data. We use Minimum Weighted Norm Interpolation (MWNI) (Liu and Sacchi, 2004), which is a Fourier reconstruction algorithm formulated as follows. The actual data, \mathbf{d} , are the result of a picking matrix (sampling), \mathbf{T} , acting on an unknown fully sampled data set, \mathbf{m} . The unknown (interpolated) data are constrained to have the same multidimensional spectrum as the original data. Enforcing this constraint requires a multidimensional Fourier Transform \mathbf{F}_{nd} . To solve for the unknown data, a cost function is defined and minimized using standard optimization techniques. The cost function J is defined as

$$J = \|\mathbf{d} - \mathbf{T}\mathbf{m}\|^2 + \lambda \|\mathbf{m}\|_{\mathbf{w}},$$

with a model norm calculated as

$$\|\mathbf{m}\| = \mathbf{m} \mathbf{F}_{nd}^{-1} |p_k|^{-2} \mathbf{F}_{nd} \mathbf{m}.$$

\mathbf{F}_{nd} is the multidimensional Fourier transform (FT) and nd is the dimension of the data. p_k is the spectrum of the unknown data, obtained by bootstrapping or iterations. All the symbols in this equation are placeholders for multidimensional matrices, but in practice they can be arranged by lexicographic ordering into long vectors. In spite of the size of these vectors, this method can be very efficient when a multidimensional Fast Fourier Transform (FFT) instead of a Discrete Fourier Transform is used. The main drawback of using the FFT is the need to bin the data along the four spatial dimensions. The potential data distortion, however, can be made negligible by reducing the 5D bin interval, working in a very fine regular grid. Although this interpolation grid usually contains many more traces than are commonly acquired in a shot/receiver consistent survey, the algorithm runs faster than working on an irregular grid with fewer traces. On the other hand, the number of unknowns in the problem grows as the bin interval becomes smaller. There is a delicate trade-off of how fine the binning should be in order to keep the position error small, versus how fine the binning can be before the proportion of original to interpolated traces becomes too small.

A consequence of working on a fine grid is that the same interpolation output can accommodate different output geometries simply by on-the-fly decimation to the target geometry. Therefore, the target geometry and the interpolation domain become almost independent of each other, making possible to design the target geometry by acquisition criteria, and interpolate the data in the domain where interpolation works better. A convenient interpolation space is the inline-crossline-offset-azimuth-frequency domain because these are the dimensions where seismic amplitudes variations are geologically meaningful. Furthermore, amplitude variations along offset and azimuth are smooth and therefore suitable for interpolation. On the other hand, wide azimuth geometries are best described in terms of shot and receiver locations because of well-understood design criteria that permit to achieve desired offset-azimuth distributions.

Numerical Advantages of binning data for Fourier Reconstruction

Simultaneous Fourier reconstruction in five dimensions for irregularly sampled data can be computationally expensive, to the point of being non practical for standard processing. By binning traces onto a fine regular grid, matrix multiplications can be replaced by FFTs and the method becomes feasible. In addition, fine binning brings two advantages for Fourier reconstruction from a computational point of view. First, it allows the algorithm to work with larger data windows than it could with exact spatial positions. Second, it helps the method to honour complex details in the data.

All interpolation methods work in a spatial “data window”, which must be large enough to include enough original traces. The sparser the sampling, the larger the window needs to be to include enough acquired information. The window size is a serious limitation for inversion algorithms for irregular data because the computation cost grows very fast with the number of dimensions. Algorithms for regular data, on the contrary, are faster and can use larger windows to interpolate very sparse data.

The second advantage of regular grids for Fourier reconstruction is that operators become quasi-orthogonal, making convergence very fast. This rapid convergence implies accurate prediction of model components with weak mapping into the Fourier spectrum, which usually have local spatial amplitude variations. Slow convergence, on the contrary, implies truncation on the solution discarding model components that have a weak mapping to the spectrum. Inversion algorithms for irregular data often suffer from slow convergence when working on large windows, producing traces with an artificial appearance that distinguish them from original traces.

Examples

The following two examples show interpolation of Foothills surveys for which shot and receiver line spacing have both been decreased by two. Figure 1 shows the shot and receiver locations and fold (color) before and after interpolation. The higher fold results in an improved offset-azimuth distribution, which improves the resulting anisotropic 3D depth migration image. Figure 2 compares the final image stack and the respective common image gathers without and with interpolation. Figure 3 shows time migration results without and with interpolation, for an area where the acquisition has a large gap because of the excavation of an open mine. Data acquired before the existence of the mine have confirmed the structure revealed by the interpolation.

Conclusions

Interpolation and data regularization play an important role in optimizing land seismic datasets for imaging. Without them, the irregular sampling and gaps commonly found in low-fold land data geometry generate migration artifacts that make the resulting images noisy. In this paper, we have discussed an efficient five dimensional interpolator based on Fourier reconstruction. By interpolating in the full seismic data space, amplitudes can realistically be predicted even in the presence of large acquisition gaps.

Acknowledgments

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References

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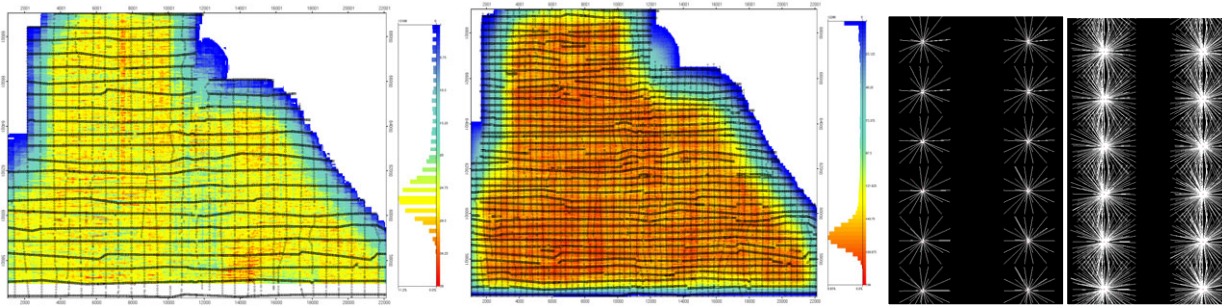


Figure 2. Shot and receiver locations before interpolation (left) and after (middle). The fold indicated by the color map is around 4 times larger after interpolation. On the right, we can see the offset azimuth distribution before and after interpolation.

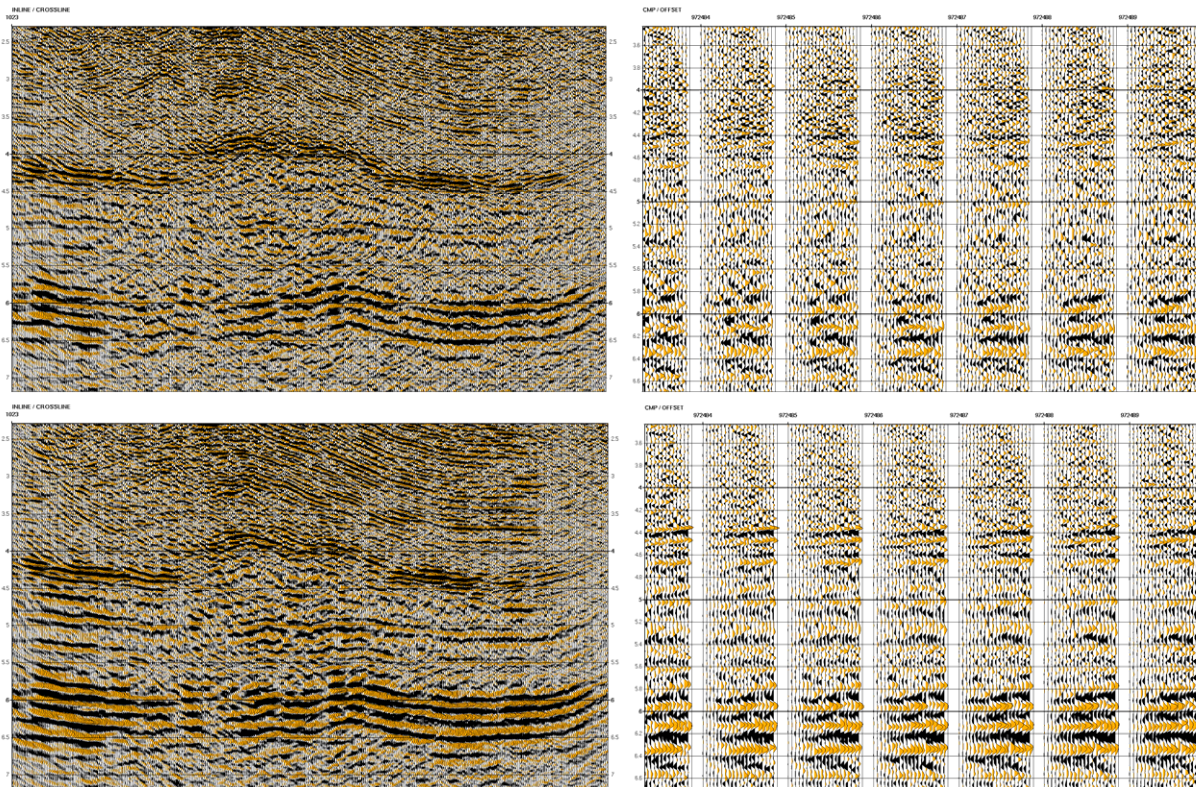


Figure 3: Benefits of prestack interpolation for anisotropic depth migration stacks and common image gathers. Above: no interpolation, Below: with interpolation before migration. Image stacks.(left) and common image gathers (right).

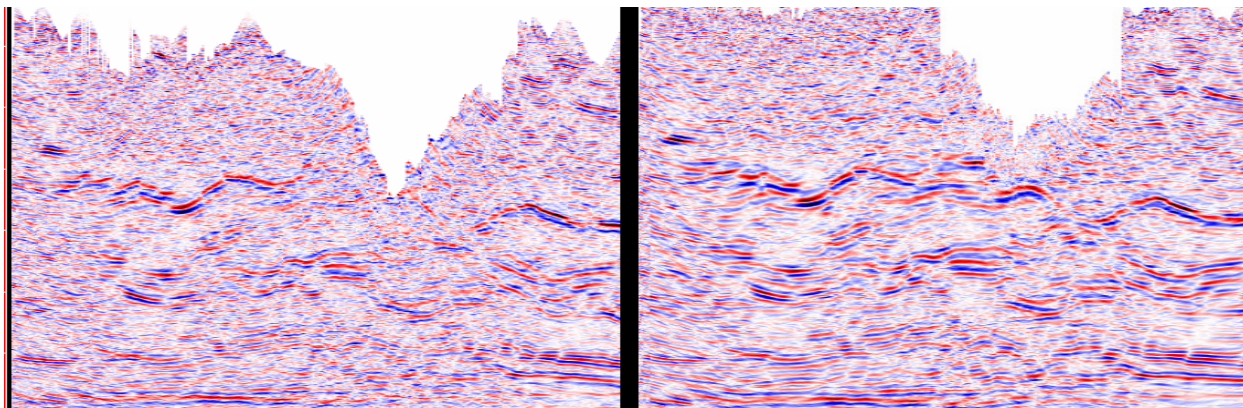


Figure 4: Time migration stack of Foothills data above an acquisition gap: without (left) and with (right) prestack interpolation before migration. The structure revealed by interpolation has been confirmed by data acquired before the existence of the gap.