Prestack Rank-Reduction-Based Noise Suppression: Theory

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Summary

Prestack random-noise suppression is an important but inadequately solved problem in land seismic processing. Two previously described techniques - eigenimage and Cadzow filtering - both use matrix-rank reduction on constant-frequency slices. These can be combined into a novel hybrid method with properties from both, forming a general class of noise suppressors that is powerful, versatile, and can be applied in any number of spatial dimensions. A companion paper in this conference shows how to apply these techniques to prestack data and gives examples.

Introduction

Some reasons for performing prestack random noise suppression are to:

- Reveal signal in noisy areas.
- Improve AVO and azimuthal analysis.
- Improve multiple attenuation, velocity analysis, and statics correction.

Such a noise suppressor should have the following properties:

- The strength can be varied easily, with the ability to make it extremely powerful.
- Handles non-uniformly spaced shooting.
- Handles both 2D and 3D data.
- Handles both plains and structured data.
- Preserves AVO and azimuthal effects.
- Preserves multiples.

Here we describe two existing methods (eigenimage and Cadzow filtering), both of which perform matrix-rank reduction on constant-frequency slices. We show how they can be hybridized to form a more general and useful class of methods with attributes of both. The companion paper "Prestack Rank-Reduction-Based Noise Suppression: Practise" (Burroughs and Trickett, 2009) demonstrate these methods them on real data.

These methods all use a noise-suppression strategy called "matrix rank reduction" (see, for example, Trickett 2003), also called truncated singular-value decomposition, principal-component analysis, subspace filtering, and many other names. The singular value decomposition, described in the previous reference, allows one to decompose a p-by-p matrix $\mathbf{A}$ into p matrices of rank one, called weighted eigenimages:

$$\mathbf{A} = \mathbf{I}_1 + \mathbf{I}_2 + \ldots + \mathbf{I}_p, \quad \left\|\mathbf{I}_i\right\|_2 \geq \left\|\mathbf{I}_{i+1}\right\|_2$$

A rank-k approximation to matrix $\mathbf{A}$ can be found by summing the first k weighted eigenimages:

$$F_k(\mathbf{A}) = \mathbf{I}_1 + \mathbf{I}_2 + \ldots + \mathbf{I}_k$$

Suppose we are given a multi-dimensional grid of traces. The general method is as follows:

Take the Discrete Fourier Transform (DFT) of each trace.

For each frequency within the signal band...

{

1. Place complex trace values for this frequency into a matrix **A** (somehow).
2. Reduce the matrix to rank *k*.
3. Recover each trace value from the matrix by averaging all elements where that value was originally placed.

}

Take the inverse DFT of each trace.

The only difference between the various techniques discussed here is how the matrix is formed in step 1.

The filter is made stronger by increasing the size of the trace grid, or decreasing the rank *k*, or both.

In practise the data set is not filtered in one piece, but rather "tiled" into overlapping blocks in both space and time, just as one would do for, say, f-x prediction. These tiles are noise suppressed separately and then recombined into a single data set by tapering and summing.

Eigenimage Filtering

Ulrych, et al (1999) described seismic applications for eigenimage filtering. Trickett (2003) applied it in the f-xy domain as follows:

Suppose have an *n*-by-*n* grid of traces. To simplify the discussion we assume square grids, but rectangular grids work just as well. For a given frequency, let the complex values from a constant-frequency slice of this grid have values $c_{i,j}, i = 1 \ldots n, j = 1 \ldots n.$ Set

$$\mathbf{A} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n,1} & c_{n,2} & \cdots & c_{n,n} \end{bmatrix}.$$

F-xy eigenimage filtering was shown (Trickett, 2003; Grimm, et al, 2003) to have the following properties:

Exactness Property*: If the grid of seismic traces is noiseless and is the sum of no more than k plane waves then f-xy eigenimage filtering with rank k does nothing to it.*

Shooting Property*: If a grid of seismic traces is noiseless and contains no more than k dips when viewed in the CMP domain then f-xy eigenimage filtering with rank k does nothing when the x-y coordinates represent common source and receiver.*

Statics Property*: F-xy eigenimage filtering is independent of x- and y-consistent statics.*

Filtering Property*: If a noiseless seismic section contains no more than k dips, and then has x- and y-consistent filters applied, then f-xy eigenimage filtering with rank k does nothing.*

None of these properties hold when eigenimage filtering is applied in the time domain, which is why the frequency domain is much preferred.

F-xy eigenimage filtering would appear to be ideal for prestack noise suppression, particularly when the x and y coordinates represent source and receiver. The exactness property suggests that it can handle dipping events as easily as flat events. The shooting property does not require that sources and receivers be evenly spaced - indeed they can be randomly positioned. The statics property suggests that this method can be applied before surface-consistent statics, and the filtering property suggests that it might be applied before surface-consistent deconvolution.

But eigenimage filtering has some problems. First, it can only easily be applied in two dimensions. Three-dimensional eigenimage is best done using multilinear rather than linear algebra - that is to say, performing rank reduction on tensors rather than matrices (De Lathauwer, et al, 2000; Wang, 2007). Multilinear algebra theory is not as elegant or straightforward as linear algebra, and we would generally like to avoid it.

Second, f-xy eigenimage is rather weak. To make it stronger one must often reduce the rank to the point of removing signal.

Cadzow Filtering

Suppose we have a one-dimensional series of n traces whose values along a constant-frequency slice are $c_i, i = 1 \ldots n.$ Perform f-x Cadzow filtering (Cadzow, 1988; Trickett, 2002) by setting

$$\mathbf{A} = \begin{bmatrix} c_1 & c_2 & \cdots & c_{n-m+1} \\ c_2 & c_3 & \cdots & c_{n-m+2} \\ \vdots & \vdots & \ddots & \vdots \\ c_m & c_{m+1} & \cdots & c_n \end{bmatrix}.$$

Variable m is usually set to half of n, making the matrix as square as possible. This is a Hankel matrix, meaning that it is constant along each antidiagonal. The exactness property holds:

If a regularly spaced sequence of noiseless traces is the sum of no more than k distinct dips then f-x Cadzow filtering with rank k does nothing to it.

Trickett (2008) extended Cadzow filtering to two spatial dimensions by forming a Hankel matrix of Hankel matrices

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \cdots & \mathbf{H}_{n-m+1} \\ \mathbf{H}_2 & \mathbf{H}_3 & \cdots & \mathbf{H}_{n-m+2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_m & \mathbf{H}_{m+1} & \cdots & \mathbf{H}_n \end{bmatrix}$$

where

$$\mathbf{H}_i = \begin{bmatrix} c_{i,1} & c_{i,2} & \cdots & c_{i,n-m+1} \\ c_{i,2} & c_{i,3} & \cdots & c_{i,n-m+2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{i,m} & c_{i,m+1} & \cdots & c_{i,n} \end{bmatrix}.$$

Cadzow filtering can be extended to any number of spatial dimensions. Three dimensions, for example, is handled by forming a Hankel matrix of Hankel matrices of Hankel matrices.

Cadzow filtering has some advantages over eigenimage. It can used in any number of spatial dimensions, as opposed to just two for eigenimage. It can also be made far stronger than eigenimage while still preserving conflicting dips.

As a rough rule of thumb, Cadzow filtering gives a four-times improvement in signal-to-noise ratio with each additional spatial dimension, assuming typical parameters are used. Thus higher-dimensional Cadzow filtering is of great interest for very noisy areas.

The exactness property holds for any number of dimensions. Cadzow filtering does not, however, have the shooting property. In particular, if the spatial coordinates represent source and receiver then sources and receivers should be uniformly spaced, a problem for land surveys. Eigenimage and Cadzow properties are summarized here:

	Eigenimage	Cadzow
Exactness Property	Yes	Yes
Shooting Property	Yes	No
Statics Property	Yes	No
Filtering Property	Yes	No
Strength	Weak	Strong
# Spatial Dimensions	2	Any

Hybrid Filtering

We can hybridize eigenimage and Cadzow filtering to derive a new type of filter that is better suited than either for prestack applications. There are at least two approaches we can take. The first is to append matrices together into larger matrices. The second is to create higher-order tensors. We shall take the first approach since it avoids multilinear rank reduction. The second approach, however, is likely the more powerful one, and may be the subject of future research.

Suppose we have two spatial dimensions, and wish to treat the first dimension as eigenimage and the second as Cadzow. Set

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 & \cdots & \mathbf{H}_n \end{bmatrix}$$

where $\mathbf{H}_i$ are the Hankel matrices from before. Since the Hankel matrices are symmetric or nearly so, it makes little difference if we append the Hankel matrices horizontally (as we've done here) or vertically. It *would* make a difference for strongly rectangular Hankel matrices.

If we have three spatial dimenions and wish to treat the first dimension as eigenimage and the other two as Cadzow then replace the Hankel matrices above with Hankel matrices of Hankel matrices. If we wish to treat two dimensions as eigenimage and one as Cadzow, set

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} & \cdots & \mathbf{H}_{1,n} \\ \mathbf{H}_{2,1} & \mathbf{H}_{2,2} & \cdots & \mathbf{H}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{n,1} & \mathbf{H}_{n,2} & \cdots & \mathbf{H}_{n,n} \end{bmatrix}$$

where $\mathbf{H}_{i,j}$ is the Hankel matrix for the *i*'th index of the first dimension and *j*'th index of the second.

In four-spatial-dimensional filtering where two dimensions are eigenimage and two are Cadzow, $\mathbf{H}_{i,j}$ is a Hankel matrix of Hankel matrices.

There are countless variations, so we propose the following notation for these hybrid filters:

E^2	Eigenimage filtering in two spatial dimensions
$\mathrm{C}, \mathrm{C}^2, \mathrm{C}^3, \mathrm{C}^4$	Cadzow filtering in one, two, three, and four spatial dimensions
EC	One dimension eigenimage, one dimension Cadzow
EC^2	One dimension eigenimage, two dimensions Cadzow
$\mathrm{E}^2\mathrm{C}$	Two dimensions eigenimage, one dimension Cadzow
EC^3	One dimension eigenimage, three dimensions Cadzow

And so on, the only limitation being that we can not have more than two eigenimage dimensions if we wish to avoid multilinear algebra.

The exactness property holds for these filters. Those dimensions that are treated as eigenimage have all the properties of eigenimage filtering. If one of the dimensions, for example, represents common source and is treated as eigenimage then we do not require source locations to be uniformly spaced and can tolerate source-consistent statics. Those dimensions treated as Cadzow, however, should be uniformly spaced and should not have large statics.

Hybrid filters fall between Cadzow and eigenimage filtering in strength. For instance, $\mathbf{C}^2$ filtering is stronger than EC which is stronger than E^2.

Conclusions

We have presented a new class of random noise suppression filters that work in any number of spatial dimensions. Individual dimensions can be customized to allow for statics or unequal spacing (eigenimage) or to maximize the strength of the overall filter (Cadzow).

These filters are well suited for removing noise from prestack seismic traces. The companion paper following shows how to apply them, with examples on real data.

References

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