A practical one way scalar wave field extrapolation method: A step forward in true amplitude processing

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Summary

A new phase-shift method for wavefield downward continuation is presented. Unlike conventional phase-shift which is based on the one way scalar wave equation that is factorized from homogenous media, this method is derived by directly solving the full scalar wave equation and therefore backward propagation is used during the procedure of extrapolation.

Introduction

The Phase-shift method (Gazdag, 1978) for wavefield extrapolation has played a very important role in exploration seismic. This method was derived by solving the wave equation in homogeneous media and it has been applied to heterogeneous media with an approximation technique for interpolation such as phase-shift plus interpolation (Gazdag, 1984) or a local wave field approximation such as generalized phase-shift plus interpolation (Wenzel, 1991). It has been shown that this one-way phase-shift migration can yield the same travel times as the full wave equation but does not yield accurate amplitudes except in homogeneous media (Kosloff and Bysal, 1983). True amplitude phase-shift migration has received attention for many years. Kosloff and Bysal developed a generalized phase-shift migration and in this work the full wavefield that contains both the up and down-going waves is used for downward continuation. However, as shown in their paper (Kosloff and Bysal, 1983) directly applying the full wavefield to phase shift migration can produce artifacts because the part of the wavefield that propagates opposite to the direction of downward continuation is not treated correctly. Therefore this method works only in a weak scattering media system. (Pai, 1988) developed a generalized *f-k* migration based on Born's approximation to handle inhomogeneous media. In this work the up going wave is eliminated when applied to the downward continuation. Because of Born's approximation this method is also limited to a weak scattering media system. Based on the WKBJ approximation (Zhang, 1993) developed a coupled equation system for up and down-going wave and this work was applied recently to true amplitude migration (Zhang *et. al*, 2005). With the same idea as that of Kosloff and Bysal, (Sharma and Agrawal, 2003) developed a finite difference based method for wide angle beam propagation and (Zhang *et al*, 2009) extended it to a split step Fast Fourier transform method for 3D wave propagation. In this paper we start from the same equation system used by Kosloff and Bysal to derive an explicit up and down going wavefield, which leads to a new one-way true amplitude phase shift downward wavefield continuation.

Methods

The scalar wave propagation problem is defined with the wave equation (2D case)

$$\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} + \frac{\omega^2}{v(x,z)^2}\psi(\omega, x, z) = 0 \qquad (1)$$

where $\psi(x,z)$ is the wavefield, x and z represent spatial coordinates and ω is temporal frequency. Equation (1) can be written as a first order ordinary equation system related to displacement and stress (Kosloff and Bysal 1983, 1987) as:

$$\frac{\partial \Phi}{\partial z} = H(x,z)\Phi(x,z) \qquad (2)$$

where

$$\Phi = \begin{bmatrix} \psi \\ \partial\psi / \partial z \end{bmatrix} \text{ and } H = \begin{bmatrix} 0 & 1 \\ -S & 0 \end{bmatrix}, \text{ where } S^2 = \partial^2 \psi / \partial x^2 + \omega^2 / v^2$$

Equation (2) is in a matrix form and can be solved numerically (Kosloff and Kessler, 1987) to obtain wavefield propagating along the spatial z direction. The solution of equation (2) for laterally homogeneous media is very simple (Kosloff and Bysal, 1983)

$$\begin{aligned} \Psi(k_x, z, \omega) &= A^+ e^{ik_z \Delta z} + A^- e^{-ik_z \Delta z} \\ \frac{\partial \psi}{\partial z}(k_x, z, \omega) &= ik_z (A^+ e^{ik_z \Delta z} - A^- e^{-ik_z \Delta z}) \end{aligned} \qquad (3)$$

where A^+ and A^-, the amplitudes of up and down-going waves, can be determined by continuity conditions of both Ψ and $\frac{\partial \Psi}{\partial z}$ at each layer boundary. Equations (2) or equation (3) are for full wave extrapolation systems and with this system one can propagate both the up and down-going wavefield simultaneously. When extrapolating the up-going wavefield the down-going reflected energy will be treated incorrectly. A possible solution to this is to separate the up and down-going wavefield at each step of the extrapolation process, store the down-going wavefield and then continue to propagate only the up-going wavefield. The stored down-going wavefield is then extrapolated in the reverse direction. This full wavefield spatial extrapolation is equivalent to reverse time wavefield propagation and needs much more memory which results in added computational cost.

In order to obtain one way wavefield extrapolation, equation (2) needs to be factorized into two equations regarding to the spatial z variable, i.e. propagating directions. The factorization can be pursuit via Eigen decomposition of matrix H:

$$H = V\Lambda V^{-1} \tag{4}$$

where V represents eigenvector and Λ is eigenvalue:

$$V = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ iS & -iS \end{bmatrix},\ \Lambda = \begin{bmatrix} iS & 0 \\ 0 & -iS \end{bmatrix} \text{ and } V^{-1} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i/S \\ 1 & -i/S \end{bmatrix}$$

by defining

$$V^{-1}\Phi(x,z) = \begin{bmatrix} U_+ \\ U_- \end{bmatrix}$$

we have

$$\frac{\partial}{\partial z}\begin{bmatrix} U_+ \\ U_- \end{bmatrix} = \Lambda\begin{bmatrix} U_+ \\ U_- \end{bmatrix} + R\begin{bmatrix} U_+ \\ U_- \end{bmatrix} \tag{5}$$

and

$$R = V^{-1}\partial V/\partial z = \frac{1}{2}\begin{bmatrix} -S_z/S & S_z/S \\ S_z/S & -S_z/S \end{bmatrix} \tag{6}$$

R is the scattering matrix and the symmetry of the matrix leads to energy conservation. Unlike equation (2), equation (5) separates the wavefield into up and down-going wavefields, but the two wavefields are coupled by matrix ***R***. When the media is homogeneous all the elements in the ***R*** matrix are zero and as a result the two wavefields are decoupled.

Equation (5) can be rewritten as:

$$\frac{\partial U_+}{\partial z} = iSU_+ + R_{11}U_+ + R_{12}U_- \tag{7}$$

$$\frac{\partial U_-}{\partial z} = -iSU_- + R_{21}U_- + R_{12}U_+ \tag{8}$$

Until now no approximation has been made. However, the coupled terms still makes the practice difficult. By ignoring the coupled terms (i.e. ignore multi-scattering) we have:

$$\frac{\partial U_{\pm}}{\partial z} = \pm iSU_{\pm} + R_{\pm}U_{\pm} \tag{9}$$

where

$$R_{\pm} = \mp\frac{1}{2}\frac{S_z}{S} \tag{10}$$

It is of interest to get an insight on the reflection term in equation (10). If $\partial^2 / \partial x^2$ can be replaced by $-k_x^2$ via a Fourier Transform as in the case of a laterally homogeneous media term then:

$$R = \frac{1}{2}\frac{v_z}{v}\frac{\omega^2}{\omega^2 - v^2 k_x^2} \tag{11}$$

and it is equivalent to the WKBJ approximation result (Zhang, 2005). Furthermore, in a layered media between layer *k* and *k+1* we have

$$R = \frac{v_k \cos\theta_{k+1} - v_{k+1}\cos\theta_k}{v_k \cos\theta_{k+1} + v_{k+1}\cos\theta_k} \tag{12}$$

which is the coefficient of reflection for an acoustic wave (Ceverny, 2001).

Applications

Extrapolators described in equation (9) can be directly solved using finite difference and can be simplified for practical implementation. With the assumption that the vertical velocity does not change within each extrapolation step and the energy at each scattering point propagates within a limited angle, due to the oblique angle effect, the reflection only happens at the boundary. Then equation (9) can be written as:

$$\frac{\partial U_{\pm}}{\partial z} = \pm isU_{\pm} + R_{\pm}U_{\pm}\delta(z - z_i) \tag{13}$$

Equation (13) can be implemented by solving the wave extrapolation within each step layer with simply a phase shift and the reflection term is only considered when moving to the next step. This implementation is similar to Pai's migration approach (Pai D., 1988) and turns out to be very efficient. For general lateral inhomogeneous media equation (13) can be solved with any conventional method such as split-step for extrapolation.

The interest point of equation (10) is the sign on the right hand. The physical explanation is that for the up-going (forward) wave extrapolation the reflection part of the energy will be removed and therefore only the remaining energy continues to propagate upwards and for the down-going (backward) propagation this lost energy will be compensated.

Discussions

A new formula for a one way equation extrapolation is presented. Comparing this formula with the conventional phase-shift method shows that they are very similar except for an extra term related to the reflection. The proposed practical implementation is similar to the amplitude treatment in ray theory. It should also be noted that the same reflection/transmission can be obtained by solving boundary problems via equation (3) and by properly eliminating the up and down-going wavefield. One more interesting point is that the scattering matrix reflects acoustic wave AVO properties and if it is extended to elastic waves then the matrix can be used for conventional elastic AVO analysis.

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