Edge-preserving seismic imaging using the total variation method

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Summary

The total variation (TV) regularization method can be used to obtain solutions were edges and discontinuities are preserved. In this article, the TV method is applied to invert acoustic perturbations using the single-scattering Born modeling operator. The TV regularization imposes sparsity on the gradient of the model parameters. The latter leads to images of model parameters with preserved discontinuities and edges. Synthetic data examples are used to test the proposed seismic imaging algorithm.

Introduction

Geophysical inverse problems are mathematically ill-posed and, therefore, regularization methods are required to obtain stable and unique solutions. In addition, regularization methods serve to impose desired features on subsurface images. Quadratic regularization methods tend to produce models where discontinuities are blurred. On other hand, non-quadratic regularization methods such as TV can provide high-resolution images of the subsurface where edges and discontinuities are properly preserved. In this paper, the total variation (TV) regularization method (Rudin et al., 1992), a non-quadratic regularization technique, is considered. In particular, we investigate its application to the problem of estimating acoustic velocity perturbations using a single-scattering Born modeling operator. In a previous contribution, Youzwishen and Sacchi (2006) proposed edge preserving regularization methods that impose sparsity on vertical and horizontal model parameter derivatives. The present contribution uses TV and sparsity is imposed on the gradient of the model parameters. This permits to better capture edges that are not horizontally or vertically aligned.

Theory and/or Method

We represent seismic data by the vector d and the model parameters by the vector m. In this case, m represents acoustic potential or velocity perturbation. Using the single-scattering Born modeling operator G, the data can be obtained via the following expression

$$d = Gm + n. \tag{1}$$

We stress that d represents the measured scattered wavefields that consist of primary arrivals. The acoustic potential can be retrieved via minimizing the following cost

$$J = \| Gm - d \|_2^2 + \mu \| \overset{\vee}{\nabla} m \|_1 \tag{2}$$

The first term is the l_2 misfit norm. This term represents the error between observations and modeled data. The second term represents the regularization term. In this case, the regularization term is the l_1 norm of the gradient of acoustic potential. The positive parameter μ is the regularization or trade-off parameter that determines the relative balance of the two terms in expression (2). The minimization of equation (2) leads to solutions where $\vec{\nabla} m$ is sparse. By promoting sparsity on $\vec{\nabla} m$, the acoustic potential becomes blocky.

Practical aspects of TV minimization

The l_1 norm is non-differentiable at 0. Therefore, our numerical implementation uses the following expression to approximate the l_1 norm of the gradient via the following differentiable functional

$$J = \| Gm - d \|_2^2 + \mu \| \sqrt{(D_x m)^2 + (D_y m)^2 + \alpha} \|_1 \tag{3}$$

where D_x and D_y are the horizontal and the vertical discrete first order derivative operators with respect to x and y, respectively. The parameter α, $0 < \alpha < 1$, is a stability parameter. The meaning and application of α will be discussed in the next section.

The gradient of equation (3) with respect to m is given by

$$\nabla_m J = G^T (Gm - d) + \mu \vec{\nabla} \bullet \frac{\vec{\nabla} m}{| \vec{\nabla} m |} \tag{4}$$

Following Dibos and Koepfler (1999) and Vogel and Oman (1998), the solution to equation (4) is found using the lagged diffusivity fixed point method. The latter permits to linearize the non-linear differential term of the right hand side of equation (4). The latter in turn leads to the solution in the following form

$$(G^T G + \mu R(m))m = G^T d \tag{5}$$

The non-quadratic regularization leads to a non-linear system of equations that can be solved using iterative methods. For instance, one can adopt the iteratively reweighted least-squares (IRLS) method. The IRLS method can be summarized with the following algorithm

$$\begin{aligned} & m^0 = 0 \\ & for\ k = 1 : \max\_iter \\ & \quad m^k = [G^T G + \mu R(m^{k-1})]^{-1} G^T d \\ & end \end{aligned} \tag{6}$$

To gain efficiency the linear system of equations in (6) can be solved with the method of conjugate gradients.

Examples

Figure 1a shows the true velocity model that we will use to test the proposed TV imaging algorithm. The vertical velocity profile is piecewise continuous, thus it contains sharp edges. First, the synthetic seismic data are generated using the known velocity reference medium. This synthetic data are assumed to be the observed data. To make the problem more realistic, noise is added to the synthetic data. Figure 1b is the generated synthetic seismic data for a single source located at position (800 m, 0 m). Figures 1c and d are the solutions obtained using quadratic regularization (least-squares with damping) and the total variation regularization method, respectively. It is evident that the TV regularization was able to recover the edges and discontinuities existing in the model.

At this point, a few words about parameter selection are in order. The solution is controlled by two parameters: α and μ. The parameter α ensures that the TV functional term is continuously differentiable. The parameter α also controls the smoothness of the solution. When α is too large the TV norm behaves like a quadratic norm and therefore, the method retrieves smooth solutions. The parameter μ is critical to find an optimal data fitting. In general μ can be found via any goodness of fit criteria. For instance, we could use the L-curve method (Hansen, 1998) or the χ^2-test (Sacchi et al., 1998).

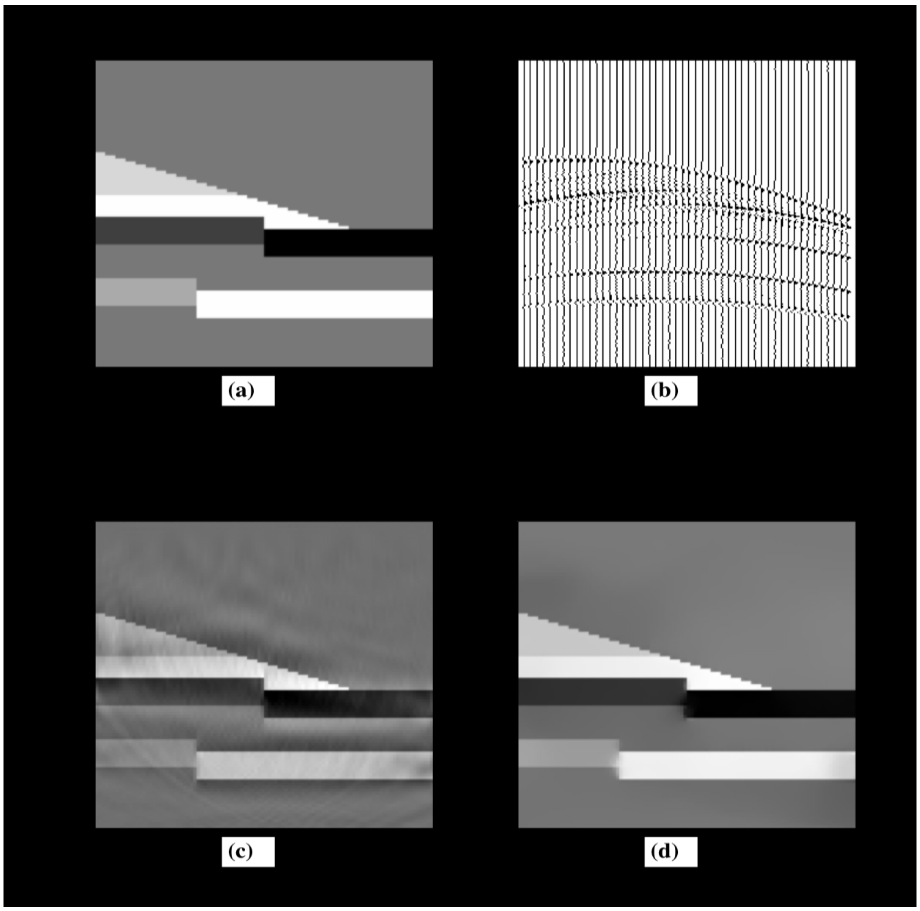

Figure 1: (a) The true layer velocity model. (b) Synthetics data obtained from the single-scattering Born approximation modeling method for a source location at 800 m from the origin. (c) Reconstructed solution using the damped least squares method. (d) Solution obtained using edge-preserving regularization implemented via the total variation (TV) method.

Discussion and Summary

The proposed imaging method uses TV regularization plus single-scattering Born modeling to retrieve a model of the subsurface with preserved edges and discontinuities. The TV norm is non-quadratic and therefore, the solution of the inverse problem requires the solution of a non-linear problem that requires solving a large inversion problem for a number of iterations. This make the problem computational intractable for large imaging pre-stack 3D problems. However, it appears that it is feasible to apply TV regularization to imaging problem that involved 2D pre-stack seismic data. The method expands some of the work proposed in the area of least-squares migration (Nemeth et al., 1999) and edge preserving imaging (Youzwishen and Sacchi, 2006).

It is clear that that TV should be used for problem that involve estimating background velocities as well. Our presentation is the first step toward the development of a non-linear inversion method that uses TV to constraint seismic velocities and not velocity perturbations.

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