

# Time Domain Least-squares Prestack Migration

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## Summary

We discuss our experience with the problem of finding the proper regularization strategy for least-squares prestack time migration. In particular, we examine quadratic regularization methods and their application to constraint the least-squares migration problem. We first review the problem of least-squares migration/inversion using asymptotic operators for pre-stack time migration (PSTM). We provide examples of regularization methods and discuss some requirements that need to be met in order to properly image subtle features.

## Introduction

Least-squares migration (LSM) was introduced as a bridge between full-wave form inversion and migration. Unlike full waveform inversion, LSM does not attempt to retrieve the background velocity model, however, like full waveform inversion the modeled data are fit to the observations. The advantage of least-squares migration is that it permits to prescribe regularization terms to impose features on the desired seismic image. For instance, smoothness in the structural image or in the prestack image gathers can be imposed in order to remove sampling artifacts (Kuehl and Sacchi, 2003). In addition, constraints can be used to increase vertical resolution (Wang and Sacchi, 2007). We are particularly interested in investigating the problem of optimal regularization methods for structural imaging. This has been recently tackled by Wang and Sacchi (2009) via FX-domain prediction error filters and Leaney et. al (2009) via dip-steering operators estimated from migrated VSP images.

## Description of the method and examples

In LSM we pose migration as a numerical optimization problem. We find an image solution  $m$  that minimizes a misfit criterion or cost function  $J=||Lm-d||^2+a.R(m)$ . In this case  $L$  denotes the de-migration operator,  $m$  the seismic image and  $d$  the prestack volume. In addition,  $R$  is the regularization term needed to stabilize the solution. In our discussion,  $m$  denotes prestack migrated gathers (un-stacked gathers). The crucial term  $R(m)$  can be a quadratic term of the form  $R(m) = |m|^2$  or a weighed norm of the form  $R(m) = |Wm|^2$ .

The cost function  $J$  is minimized using the method of conjugate gradients. If the damping term is omitted ( $a=0$ ), one can show that the solution converges to the minimum norm solution. Figure 1a portrays a reflectivity model used to test our algorithm. Figure 1b is the image obtained by NMO followed by stacking the gathers produced with the finite difference method from the reflectivity model in Figure 1a. Figure 2a is the PSTM image obtained by standard pre-stack time domain Kirchhoff migration. The minimum norm solution associated to this experiment is

displayed in Figure 2b. The amplitudes in the LSM solution are preserved with good degree of resemblance to the original reflectivity model. In the next experiment, we introduce an operator  $W$  in the LSM algorithm. In this case,  $W$  entails performing local derivative along the structural dip in conjunction with second derivative along offset in image space. This is to penalize roughness along off-dip directions and along common image gathers. Figure 3 is used to compare the solution with minimum norm regularization with that using a priori dip information. We notice, as expected, that the method fail to properly image the data in areas with conflicting dips. This is a consequence of using only the most energetic local dip to build the operator  $W$ . We clearly see that a proper regularization strategy will require a mean to switch the regularization off in areas of conflicting dips. This can be achieved via introducing non-quadratic norms (Youzwishen and Sacchi, 200X). Unfortunately, non-quadratic norms will lead to non-linear optimization problems and to an increased computational cost.

## Discussion

We have developed an algorithm for least squares PSTM. The algorithm offers the possibility to incorporate a variety of constraints including dip-dependent regularization. Our preliminary results indicate that dip-based regularization can help in areas of low structural complexity. In particular, when trying to image badly sampled data. High resolution imaging of subtle features like fault planes (as indicated in our examples) will require utilizing more complex regularization strategies capable of de-emphasizing smoothing along areas of incongruent dips.

## References

- Leaney W.S., M.D. Sacchi, and T.J. Ulrych, 2009, Least-squares migration with dip, SEG, Expanded Abstracts, 28, 2864.
- Kuehl H. and Sacchi M.D., 2003, Least-squares wave-equation migration for AVP/AVA inversion: Geophysics, 68, 262-273.
- Youzwishen C. F. and Sacchi M.D. 2006, Edge Preserving Imaging, Journal of Seismic Exploration, Volume 15, 4, 45-58.
- Wang J. and Sacchi M.D., 2007, High-resolution wave equation AVP imaging with sparseness constraints, Geophysics, 72 (1), S11-S18.
- Wang J. and M.D. Sacchi, 2009, Structure constrained least-squares migration, SEG, Expanded Abstracts, 28, 2763.

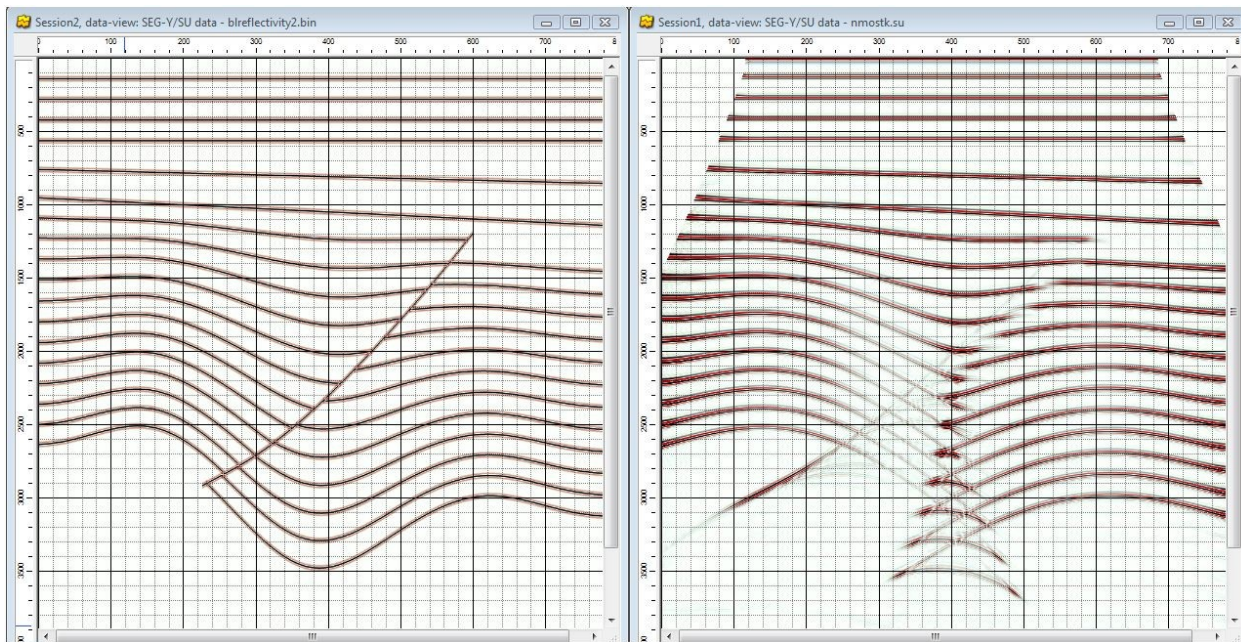


Figure 1. a) Structural model created to test out PSTM algorithm. b) Stacked section created by NMO followed by stacking.

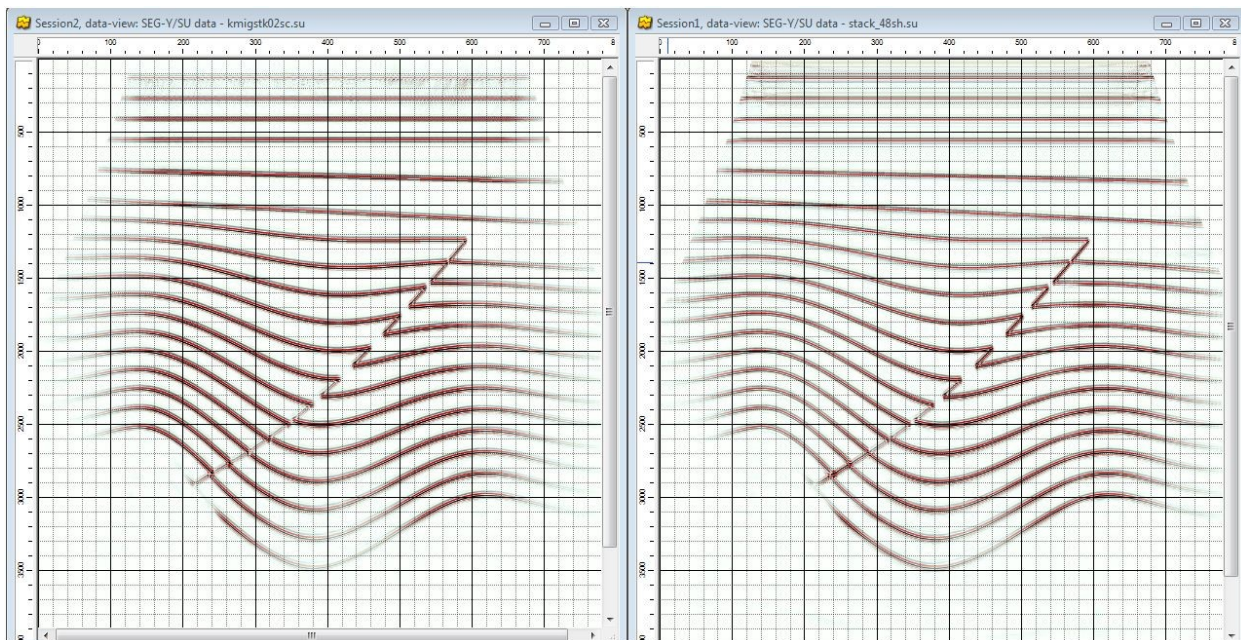


Figure 2. a) Prestack Kirchhoff migration. b) Un-regularized (minimum norm solution) prestack time least squares migration.

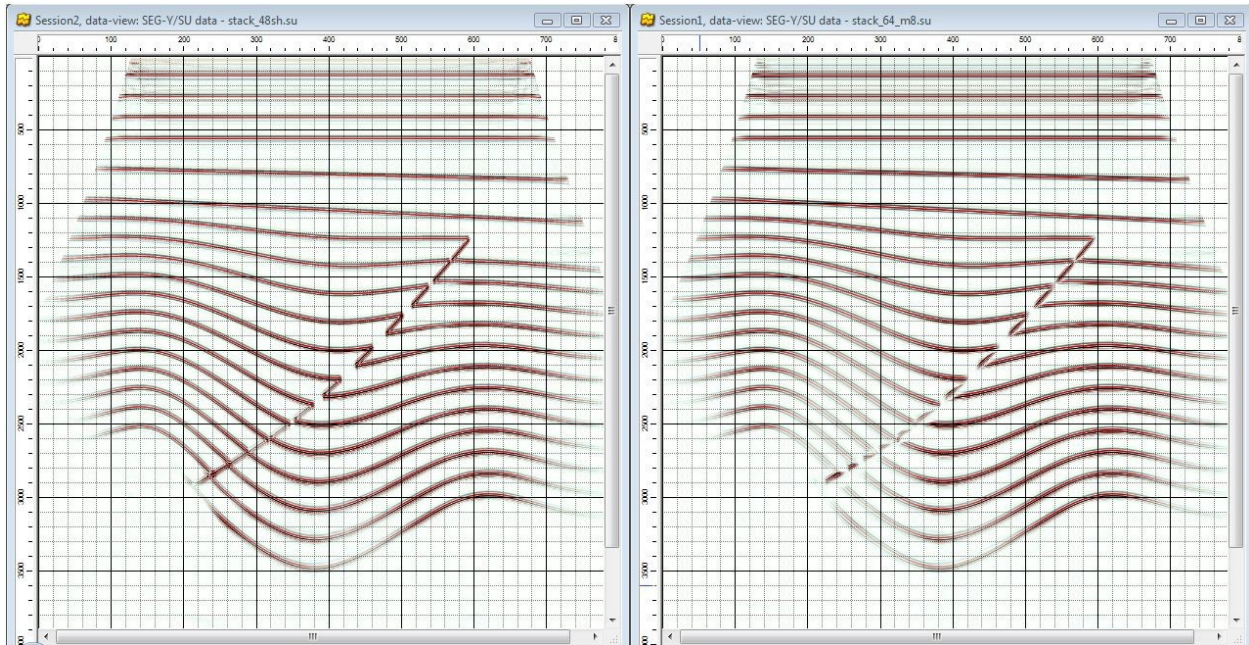


Figure 3. a) Un-regularized prestack least squares migration (minimum norm solution). b) Local dip-dependent derivative regularization.