Azimuthal Simultaneous Elastic Inversion

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Summary

Azimuthal AVO has proved to be useful to predict fractures. However, there are a number of limitations with the technique including the fact that only band limited fractional elastic parameters are estimated. Further, the method is derived for the case of an isotropic over HTI anisotropic half-space. It is not theoretically valid for the case of two anisotropic half-spaces. In order to overcome these limitations this paper develops and demonstrates a new azimuthal simultaneous elastic inversion. Azimuth- and angle-limited seismic traces are modeled using the anisotropic Zoeppritz equation to generate equivalent reflectivity volumes which are then convolved with appropriate user defined wavelets, generating 3D volumes of the fracture parameters. HTI anisotropy, the lowest symmetry which describes fractures, is used in this implementation. Alternative parameterizations using rock physics models are examined to reduce and constrain the parameterization. The algorithm is demonstrated on both synthetic and real seismic data with good results.

Introduction

The estimation of HTI anisotropy has proved useful to predict fractures (Hunt et al., 2010) and horizontal stress (Gray, 2010). One proven method to predict HTI anisotropy is azimuthal AVO. Rüger (2002) derived a linearized approximation to the Zoeppritz equation for HTI anisotropy. The near offset approximation of this is similar to the two-term Shuey AVO equation with two extra parameters, the anisotropic gradient and the symmetry plane. For fractured media, the anisotropic gradient is often claimed to be proportional to crack density while the isotropy plane is thought to be the strike of the fractures. There are a number of assumptions and limitations with this. First, the derivation assumes an isotropic half-space over an anisotropic half-space. This assumption is restrictive as we would like to generalize the model to the case of a stack of anisotropic layers. Secondly, as Goodway et al. (2006) argue, the near offset approximation is susceptible to theoretical error introduced by the far offset terms. In addition, the anisotropic gradient is actually a function of Thomsen's anisotropic parameters delta and gamma. These two parameters may not be correlated in the same fashion to crack density giving rise to potentially a complex relation between crack density and the anisotropic gradient. Further, there is a 90 degree ambiguity associated with the estimate of the isotropy plane (Rüger, 2002). Lastly the azimuthal AVO inversion estimates only fractional band limited elastic parameters. Simultaneous prestack elastic inversion is one way to address these limitations. Coulon et al. (2006) demonstrated a simultaneous prestack inversion to invert for isotropic elastic parameters using a simulated annealing algorithm. This paper extends this approach to anisotropic media and by so doing addresses the limitations outlined above.

This paper first reviews the 3D, simultaneous isotropic elastic inversion of Coulon et al. (2006). The isotropic simultaneous inversion uses a 1D convolutional modeling scheme where the reflectivity is modeled by the Zoeppritz equation or some linearization of that. There is then a discussion about how the reflectivity modeling is generalized to the case of two orthorhombic half-spaces with arbitrary rotated symmetry planes following Schoenberg and Protázio (1992). Even though the reflectivity modeling supports orthorhombic anisotropy the more restricted case of HTI anisotropy is assumed to reduce the number of free parameters solved for. Further, alternative parameterizations incorporating fractured rock physics such as the penny-shaped crack model (Hudson et al., 1981) or linear slip deformation (LSD) theory (Schoenberg and Sayers, 1995) are explored. After describing the calculation of the anisotropic Zoeppritz equation and the parameterization of this, the generalization of the Coulon et al. (2006) method to anisotropic media is described. The method is then demonstrated on both synthetic and real data.

Theory

Coulon et al. (2006) describe a 3D multi-cube simultaneous isotropic elastic inversion. The inversion is 3D in the sense that it solves for a 3D parameter volume but in reality models the data with a 1D convolutional modeling scheme. The reflectivity is modeled using the Zoeppritz equation or some linearization of that. The inputs to the algorithm are angle stacks and some initial layered elastic model defined in the time domain. By using angle stacks, NMO stretch (Roy et al. 2005) and scaling issues can be addressed by varying the wavelet as a function of angle of incidence. Further, ray tracing need not be performed, simplifying the forward modeling. The initial model is iteratively perturbed using simulated annealing to find a global solution which minimizes the objective function. The objective function contains a data misfit and regularization term. The regularization term includes a 3D spatial continuity constraint to help attenuate the effects of random noise. Further, since the algorithm is nonlinear, bounds may be easily incorporated. The algorithm perturbs the layer P-wave velocity, V_p, S-wave velocity, V_s, and density, ρ. These parameters can be perturbed independently or coupled via relationships such as the Gardner's relation linking V_p and ρ. In addition to the elastic parameters, the method also perturbs the time-thickness of the micro-layers so as to reduce the data misfit and enhance lateral coherence.

In order to extend this algorithm to anisotropic media the reflectivity calculation needs to be modified to incorporate anisotropic media. The HTI layered medium may be parameterized in terms of the layer time-thickness, P-wave and S-wave impedances, density, and the Thomsen parameters delta, epsilon and gamma and the azimuth of the isotropy plane. This gives 8 free parameters per layer. The HTI stiffness matrix is calculated from the elastic parameters defining each layer. Then the isotropy plane information is used to perform a Bond transformation (Winterstein, 1990). This formulation allows the isotropy plane to vary as a function of layer or equivalently depth. Schoenberg and Protázio (1992) solve for the Zoeppritz reflectivity using the rotated stiffness matrices as input. The reflectivity is modeled for each interface resulting in a reflectivity series. This reflectivity series is then convolved with some user defined wavelet to create a model of the data for a particular azimuth and angle of incidence. The simultaneous inversion methodology of Coulon et al. (2006) using simulated annealing extends naturally to the nonlinear forward modeling described above.

The introduction of HTI anisotropy introduces four additional parameters to the four parameters of the isotropic inverse problem (i.e. three Thomsen parameters and azimuth of the isotropy plane). This raises the question whether the problem is well enough posed to obtain a reliable estimate of all these parameters. It is possible to reduce the number of free parameters solved for by making use of rock physics models. The LSD theory of Schoenberg and Sayers (1995) reduces the number of parameters describing the HTI stiffness matrix by one. In this theory the stiffness matrix is described by the isotropic parameters λ and μ, and the normal and tangential weakness δ_N and δ_T. These weakness parameters describe how fractures weaken a background isotropic rock. The penny-shaped crack model of Hudson et al. (1981) provides an alternative way to parameterize the model space. It can be related to LSD theory with the following relations

$$\delta_T = 16\varepsilon/(3(3-2g)), \tag{1}$$

$$\delta_N = \frac{4\varepsilon}{3g(1-g)} \Big/ \left(1 + \frac{1}{g}\frac{\varsigma}{1-g}\right), \tag{2}$$

where g is the square of the unfractured background rock V_s/V_p ratio, ε is the crack density, and ς is related to the fluid and aperture, a, by the relation

$$\varsigma = \frac{1}{\pi a}\frac{\rho_{liq}V_{p_{liq}}^2}{\rho V_p^2}. \tag{3}$$

If the fluid is known *a priori* to be gas, such as in the Deep Basin, the unknown anisotropic parameters can be reduced to just the crack density and isotropy plane azimuth.

Having extended the forward modeling to anisotropic media and developed suitable parameterizations, it is now possible to describe how the simultaneous inversion methodology of Coulon et al. (2006) is extended to anisotropic media. The inputs to the algorithm are still angles stacks but now specified at a variety of different azimuths. In the real data example four angle stacks are used and six azimuths (i.e. 15, 45, 75, 105, 135 and 165 degrees) resulting in 24 input cubes for the inversion. The input angle-azimuth stacks are created using a controlled amplitude processing flow outlined by Gray et al. (2009). In addition to the isotropic initial layered model, the user must also specify the anisotropic model. Our algorithm assumes the fractures follow a power law or fractal relationship with the isotropy azimuth governed by the regional stress regime.

The simultaneous inversion is similar to the isotropic inversion but with the incorporation of azimuthal effects. The initial model is iteratively perturbed using a simulated annealing algorithm to minimize the objective function. The data misfit portion of the objective function minimizes the differences between the anisotropic forward modeling described above and the angle-azimuth cubes. The regularization term in the objective function once again optimizes 3D spatial continuity. The perturbations may be applied to the individual parameters or perturbations can be coupled via correlations between the parameters. Both the LSD and Hudson parameterizations couple the Thomsen parameters and limit the solution space.

Results

The algorithm was tested on both real and synthetic data. Figure 1 shows the input parameters used to generate synthetic data for two tests that were performed. Both the second and third layers are anisotropic with different symmetry planes. This case breaks the assumptions made by the Rüger (2002) equation. The data was forward modeled using the anisotropic Zoeppritz equation generating reflectivity for angle of incidence stacks at 10, 20, 30 and 40 degrees and for azimuths at 0, 30, 60, 90, 120, and 150 degrees. These angle-azimuth stacks were then convolved with a wavelet. Data was generated for a 3D volume where the parameters were held laterally invariant. The synthetic seismic data were then inverted using the simultaneous anisotropic inversion. Figure 2 shows the anisotropic parameter estimates for the third layer displayed as a probability distribution (PDF). The ideal solution is highlighted on the respective axes with a red box. The match is excellent with only a small amount of scatter about the ideal solution.

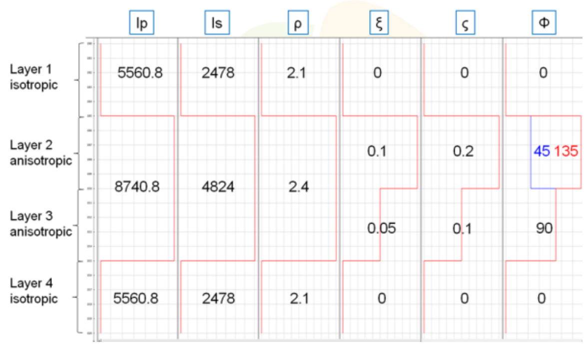

Figure 1. The input model used to generate the synthetic seismic data. Two different models were generated one with the second layer having a 45 degree symmetry axis and the other with the second layer having a 135 symmetry axis.

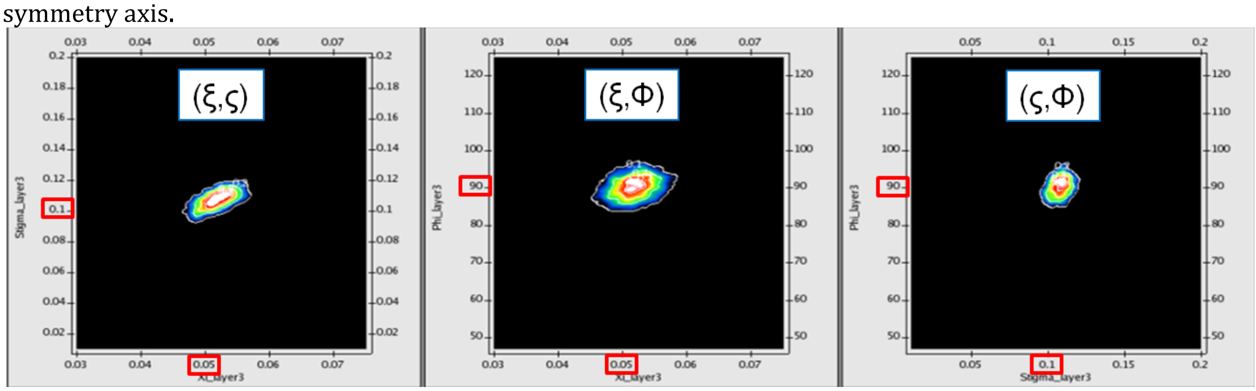

Figure 2. Cross-plot showing parameter estimates for crack density, ε, fluid term, ς, and symmetry axis, ϕ. The figures a), b) and c) are 2D projections of the 3D solution space.

The model in Figure 1 shows two different models for the second layer, one with a 45 degree symmetry axis and the other with a 135 degree symmetry axis. This was done to test whether the algorithm resolved the 90 degree symmetry plane ambiguity that the Rüger (2002) equation has. The inversion gave correct results in both cases.

The inversion was tried on a 3D seismic dataset from North-Eastern British Columbia. The estimate of the normal and tangential weaknesses δ_N and δ_T for the real seismic dataset are shown in Figure 3.

Recall that in fractured media the tangential weakness is proportional to the crack density while the normal weakness is also a function of the crack aperture and fluid (equation 2 and 3).

Conclusions

In summary, we have developed and demonstrated a new 3D simultaneous elastic inversion for HTI anisotropic media. The method is an extension of the isotropic simultaneous inversion of Coulon et al. (2006). The method addresses a number of theoretical shortcomings of azimuthal AVO. Rather than producing fractional elastic parameter estimates, the inversion produces elastic parameter estimates. The method is general enough to allow each layer to be HTI anisotropic with an arbitrary rotation for the symmetry axis. The forward modeling calculates the reflectivity using the anisotropic Zoeppritz equation or some linearization of this and does not rely on some near offset approximation to enhance stability. Lastly, the 90 degree symmetry axis ambiguity has been removed. The algorithm was demonstrated on both synthetic and real data with good results.

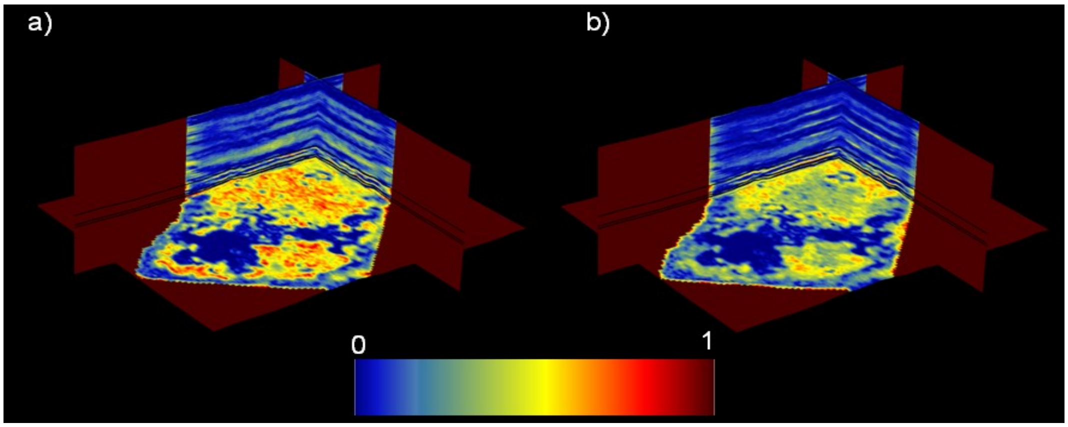

Figure 3. The estimate of the normal and tangential weaknesses δ_N a) and δ_T b) for the real seismic dataset.

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