recovery energy environment economy
2011 CSPG CSEG CWLS Convention

A New f-p Domain Weighting Function in Minimum Weighted Norm Interpolation of Seismic Data

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Summary

Seismic trace interpolation can be formulated as an underdetermined least squares inverse problem. In order to force interpolation to follow the directions of seismic events, the weighting function is designed to build the interpolated energy along the desired direction. Usually, such a weighting function is built in the Fourier domain. However, when Fourier spectra are aliased, especially in the case of up-sampling seismic traces (creating more output than input), the design of the weighting function faces problems. In this paper, we discuss these problems and a proposed solution using the f-p domain.

Introduction

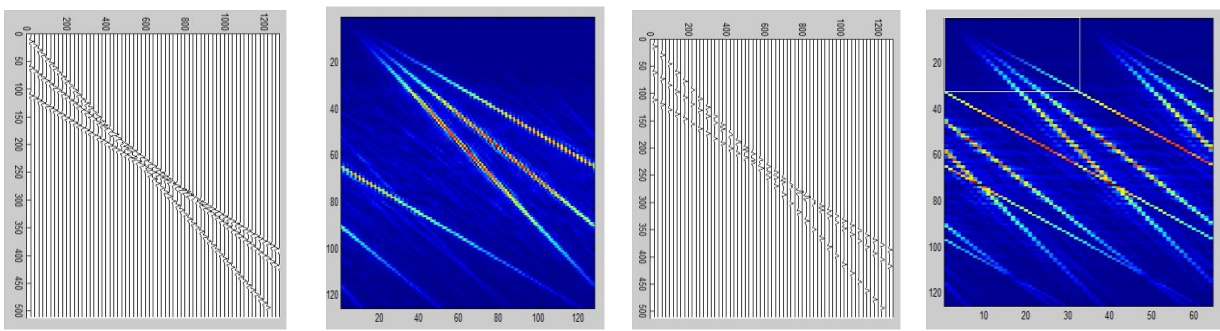
Seismic trace interpolation or reconstruction can be formulated as an underdetermined inverse problem. In order to let interpolation follow the underlying seismic events (rather than the noise events), a weighting function is employed as a constraint. Such algorithms are usually referred to as Minimum Weighted Norm Interpolation (MWNI) (e.g. Liu and Sacchi, 2004). Unlike Spitz's approach (1991), MWNI does not require the assumption of linear events and can be easily extended to un-even trace interval cases. It has therefore gained great interest. Usually, when filling random missing traces, the f-k domain is used to build the weighting function. The advantage is that the pattern of energy distribution, i.e. the underlying seismic events, shows a high covariance between interpolated and un-interpolated data in the f-k domain as opposed to the original time-space domain. This guarantees that the pattern of the f-k spectra after interpolation is unchanged. When interpolation is applied to the case where traces need to be denser (e.g. adding traces between two neighbouring traces), the spectrum in the f-k domain before interpolation is aliased due to the reduced effective Nyquist interval, and therefore high spectrum covariance no longer exists. In this case, if we still want to use the f-k domain for building weighting functions, the aliased portion of the spectrum must be removed first.

However, removing the aliased portion of spectra has been shown to be a difficult task. Several algorithms have been developed for solving the problem (e.g. Schonewille and Vigner, 2009, Yao, 2010). Similar to Spitz's approach, these algorithms are all based on the assumption of linear events and the un-aliased portion of spatial spectra at low temporal frequency can be linearly extrapolated to that at high temporal frequency. This assumption shrinks the application range of the original MWNI method because the linear assumption may not always satisfy the real case.

For these reasons it is advantageous to find another dual domain where aliasing does not appear and the underlying characteristics of seismic events in the f-k domain are preserved. (A "dual" domain refers to the data in either of two domains (e.g. time-space and FK) where the data can be converted from one domain to the other by some transform – e.g. a double Fourier Transform). In this paper, we will describe the f-p domain approach and show its advantages over the f-k domain.

Graphical presentation of low frequency extrapolation in f-k domain

The traces to be interpolated are treated as missing traces to be filled. When such missing traces are regular, the effective spatial Nyquist interval is reduced. This leads to aliasing of the FK spectrum. For instance, as shown in Figure 1, the missing traces version of the original spectrum is aliased by extra copies of the original spectrum.



(a) (b) (c) (d)

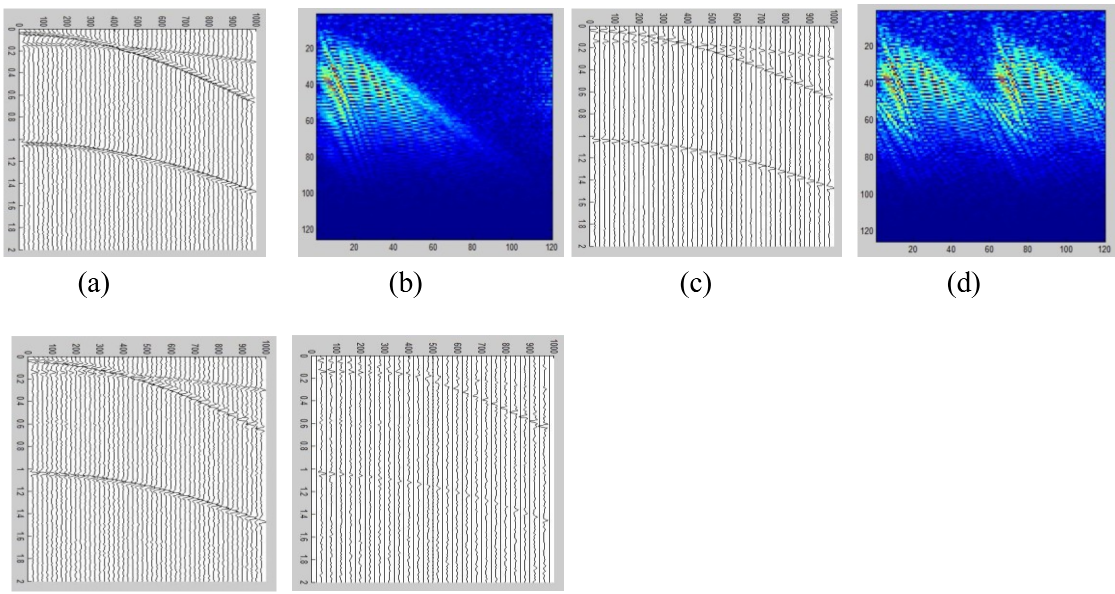
Figure 1. Synthetic data and its f-k spectra (a) and (b) for original data; (c) and (d) for missing traces data.

Because the weighting function with spectra in Figure 1d should use a similar energy pattern as that in Figure 1b, the copied (aliased) spectra need to be avoided. In the case of linear events as shown in Figure 1, because the pattern of energy is simply a linear function and all the information in a spectral portion at high temporal frequency can be obtained from a spectral portion at low frequency, the avoidance can be realized by simply linearly extrapolating the portion within the rectangle (top left) to the whole FK space.

However, when events are curved, linear extrapolation is no longer valid. As an example, Figure 2a shows synthetic data with curved events with Figure 2b as its FK spectrum. If we decimate (remove every second trace) we get the data shown in Figure 2c, with the aliased FK spectrum of 2c shown in Figure 2d. Simply investigating the figure can conclude that spectra are by no means a linear function of temporal frequency. Using the Spitz method for creating/interpolating every other missing trace, the output is in 2e and the difference is in 2f. The Spitz result shows that the linear assumption required cannot handle curved events well, which means that low frequency extrapolation as described above for Figure 1d is not valid for curved events.

f-p domain approach

We would like to have a dual domain where the pattern of energy distribution does not change when traces are missing. It is worth noticing that methods that try to "avoid" aliased energy in the f-k domain are actually using some properties of the Radon domain implicitly: the low frequency extrapolation technique assumes that the dip angles of the peak energy in f-k are independent of frequency and these dipping angles themselves are nothing but p parameters in the Radon domain. Hence we will now choose the Frequency Radon domain, i.e. f-p domain, as the dual domain for building weighting functions. By "Radon" we mean any of the standard forms whether it will be linear, parabolic or some other form. The great advantage of the f-p domain is that the copied portions of the spectra (aliased energy) no longer appear in this domain and for linear events, the pattern of energy distribution is, instead of a dipping line in the f-k domain, a series of vertical lines that lie parallel to the frequency axis that can be more easily manipulated as, for example, when smoothing the data. Moreover, the parabolic Radon domain can take curved events into account.



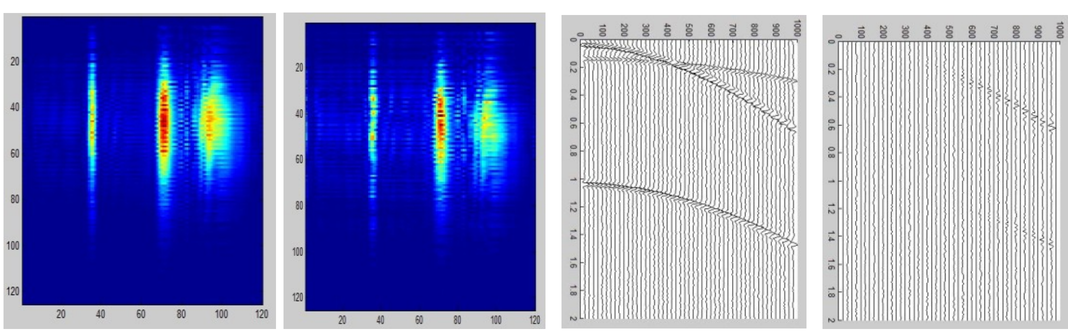
(a) (b) (c) (d)

(e) (f)

Figure 2. (a) and (c) is synthetic data and its decimation of every other trace; (b) and (d) are corresponding spectra; (e) is interpolated result with data and (f) its difference from (a).

Example

Synthetic data is the same as that in Figure 2 and corresponding to Figure 2a and 2c, the f-p spectra, with parabolic radon transform, are in Figures 3a and 3b, which show similar patterns of energy between the original and the "missing traces" data. In 3a, comparing with Figure 2b the f-p spectra shows a much simpler pattern that clearly represents the events in the data. With the weighting function built in this domain, the interpolated output is shown in 3c and difference is in 3d.



(a) (b) (c) (d)

Figure 3. f-p spectra for (a) corresponding to Figure 2a and (b) corresponding to Figure 2c via parabolic radon transform; (c) and (d) are result and difference corresponding to Figures 2e and 2f.

Conclusions

An approach that builds weight functions in the f-p domain has been proposed for Minimum Norm Weighted Interpolation. With the proper choice of the p parameter (e.g. linear, parabolic etc), the alias problem that is inherent in the f-k domain can be avoided. Moreover, the f-p domain can be more flexible for the interpolation of curved events.

References

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