Multi-trace Acoustic-impedance Inversion with Transform-domain Sparsity Promotion

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Summary

We propose a multi-trace acoustic-impedance inversion method based on transform-domain sparsity promotion in this paper. With a priori information that the impedance model updates are sparse in transform-domain, we firstly invert for sparsifying transform coefficients, and then transform them back to physical (time-space) domain. Thus, multi-trace impedance updates within 2-D section or 3-D subvolume can be obtained. An example is used to test our method.

Introduction

Seismic inversion is a key tool to estimate impedance parameters from seismic data (Morozov and Ma, 2009; Veeken et al., 2009). However, it is difficult to invert for true impedance since conventional acoustic-impedance inversion is ill-posed. Therefore, priori information, such as sparse reflectivity or blocky impedance assumptions (Oldenburg, Scheuer and Levy, 1983), is often added to partially fill in the null space of ill-posed time-domain acoustic-impedance inversion.

Recently, a new type of priori information with transform-domain sparsity promotion, such as curvelets, has been proposed and applied in seismic forward modeling (Herrmann, Erlangga and Lin, 2009), data processing (Herrmann, Boeniger and Verschuur, 2007; Herrmann and Hennenfent, 2008; Herrmann and Li, 2011) and full-waveform inversion (Li et al., 2011). This priori information regards the inversion parameters or model updates as sparse or compressible in the curvelet domain (Herrmann, Moghaddam and Stolk, 2008), thus providing efficient representations of parameters or models that are smooth except for discontinuities/wavefronts along piece-wise smooth curves.

In this paper, we propose a multi-trace acoustic-impedance inversion method with transform-domain sparsity promotion. The multi-trace acoustic-impedance parameters are stably solved by a modified generalized linear inversion (GLI) method, which replaces the linearized subproblem (Cooke and Schneider, 1983) with basis pursuit de-noise (BPDN) problem (van den Berg and Friedlander, 2008). Our method transforms the model updates in the physical domain to curvelet coefficients in scale-position-angle domain. By inverting for the curvelet coefficients, followed by applying the inverse, we obtain model updates for each iteration.

Method

For the post-stack impedance inversion problem, the calculated seismic data can be represented by

$$d^{cal}(t) = w(t) * \sum_{i=1}^{M} \frac{m_{i+1} - m_i}{m_{i+1} + m_i} \delta(t - T_i) \text{ , (1)}$$

where *w*(*t*) represents seismic wavelet, and m_i represents acoustic-impedance at time T_i.

For every single observed trace, acoustic-impedance can be estimated by minimizing the following objective function

$$O(\mathbf{m}_l) = \| \mathbf{d}_l^{obs} - \mathbf{d}_l^{cal} \|_2^2 \text{, (2)}$$

where $l \in \{1, 2, ..., N\}$, *N* is the total number of traces, $\mathbf{d}_l^{obs}$ and $\mathbf{d}_l^{cal}$ represent the *l*-th trace of observed and calculated seismic data, respectively.

For simplicity, we assume that the seismic wavelet is known. By neglecting high-order terms, the Taylor expansion of $\mathbf{d}_l^{cal}$ at $\mathbf{m}_{l,0}$ is written as

$$\mathbf{d}_l^{cal}(\mathbf{m}_l) \approx \mathbf{d}_l^{cal}(\mathbf{m}_{l,0}) + \left. \frac{\partial \mathbf{d}_l^{cal}(\mathbf{m}_l)}{\partial \mathbf{m}_l} \right|_{\mathbf{m}_{l,0}} \left(\mathbf{m}_l - \mathbf{m}_{l,0} \right) \text{. (3)}$$

Assume that the sum of high-order terms of the Taylor expansion and noise in the observed data is $\mathbf{n}_l$, the following equation can be obtained

$$\Delta \mathbf{d}_l = \mathbf{d}_l^{obs}(\mathbf{m}_l) - \mathbf{d}_l^{cal}(\mathbf{m}_{l,0}) = \mathbf{J}_l \Delta \mathbf{m}_l + \mathbf{n}_l \text{, (4)}$$

where $\mathbf{J}_l = \left. \frac{\partial \mathbf{d}_l^{cal}(\mathbf{m}_l)}{\partial \mathbf{m}_l} \right|_{\mathbf{m}_{l,0}}$ is the Jacobian matrix, which is always non-invertible since the seismic wavelet is band-limited, and $\Delta \mathbf{m}_l = \mathbf{m}_l - \mathbf{m}_{l,0}$ represents the model update.

To construct multi-trace simultaneous inversion and adopt sparsity promotion, we rearrange equation 4 as the following matrix/vector equation

$$\Delta \mathbf{d} = \mathbf{J} \mathbf{P}^H \mathbf{x} + \mathbf{n} \text{, (5)}$$

where the model vector is given by $\mathbf{P}^H \mathbf{x} = \left[\Delta \mathbf{m}_1, \Delta \mathbf{m}_2, \cdots, \Delta \mathbf{m}_N \right]^T = \Delta \mathbf{m}$, $\mathbf{P}$ is a sparsifying transform operator, $\mathbf{x}$ is the transform coefficient vector, $\Delta \mathbf{d} = \left[\Delta \mathbf{d}_1, \Delta \mathbf{d}_2, \cdots, \Delta \mathbf{d}_N \right]^T$ is the data vector, and $\mathbf{J}$ is a blocky diagonal matrix with diagonal elements $\mathbf{J}_1$, $\mathbf{J}_2$, …, $\mathbf{J}_N$. Given curvelet transform operator $\mathbf{P}$, we calculate $\mathbf{x}$ by solving the basis pursuit de-noise (BPDN) problem

$$\min_{\mathbf{x}} \| \mathbf{x} \|_1 \quad \text{s.t. } \| \Delta \mathbf{d} - \mathbf{J} \mathbf{P}^H \mathbf{x} \|_2^2 \leq \sigma \text{, (6)}$$

with an appropriately chosen tolerance σ.

After $\mathbf{x}$ is estimated, we have the current iteration result

$$\mathbf{m}_1 = \mathbf{m}_0 + \mu \mathbf{P}^H \mathbf{x}, \quad (7)$$

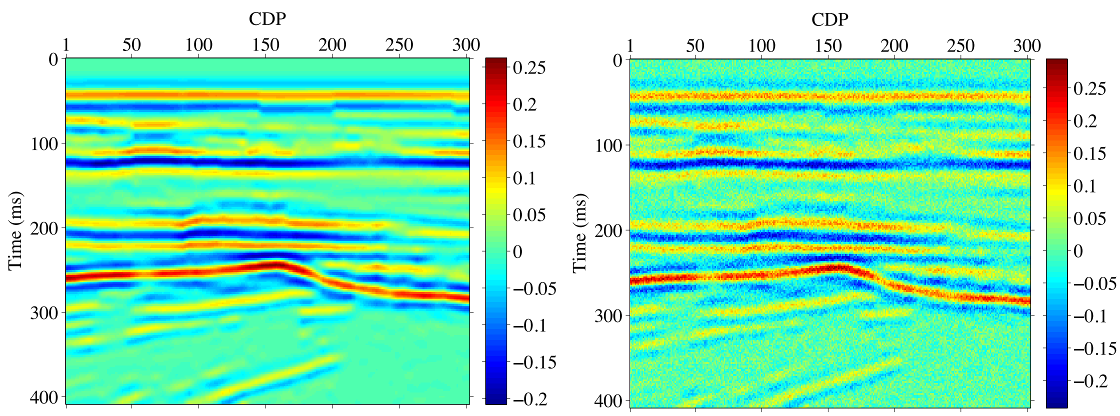
where μ is the step length.

Substituting the result of equation 7 into equation 1 and repeating solving equation 6 and equation 7, we get the next iteration result. Once the termination condition meets, we obtain the optimized multi-trace acoustic-impedance result.

Examples

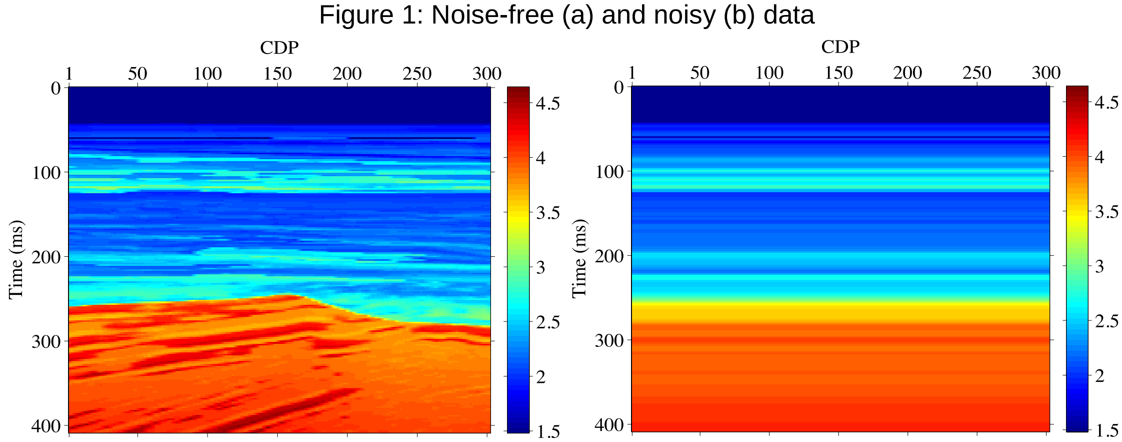
To test the method, we generate a noise-free observed data (figure 1a) by convoluting a 30-Hz Ricker wavelet with the reflectivity derived from a synthetic BG impedance model (figure 2a), consisting of 302 traces with a sample interval of 2 ms. Figure 1b is the noisy observed data by adding 30% (NSR = 0.3, defined as the ratio of noise energy to signal energy) random noise to the noise-free observed data.

For the two observed data, an initial model (figure 2b) without lateral variation information is provided, and all traces are used to implement the inversions. After six updates for impedance model, we get the inversion result (figure 2c) for noise-free data. As figures 2a-2c show, the method reconstructs spatial geology structures and invert for impedance close to the true model. After five updates, we get the inversion result (figure 2d) for noisy data. As figures 2a and 2d show, although the observed data contains noise, especially for high-frequency from 70 Hz to 250 Hz, the inversion result still has a good consistency with the true impedance model.

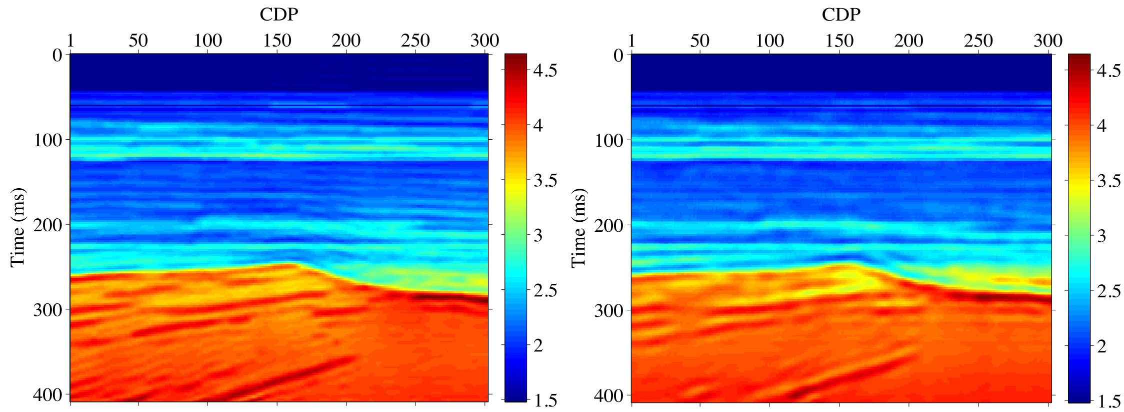


(a) (b)

Figure 1: Noise-free (a) and noisy (b) data



(a) (b)



(c) (d)

Figure 2: True impedance model (a), initial impedance model (b), and inversion results for noise-free data (c) and noisy data (d)

Conclusions

The multi-trace simultaneous inversion method we proposed in this paper exploits natural sparsity of model updates in the curvelet domain, which acts as a powerful prior regularizing the inversion of the linearized subproblems. The method is robust with respect to random noise. Whether the method exploits spatial continuity of seismic data by using transform-domain sparsity promotion is our next topic.

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