

# Pseudo ARMA FX deconvolution for seismic data noise reduction

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### Summary

Among the many denoising methods developed in recent years, FX filter is one of the most powerful algorithms that is used in daily seismic data processing. The conventional FX filter is actually a convolution filter with its convolution operator generated by an AR model. While reducing noise, this convolution operator can also smooth out some of the detailed information embedded in seismic data to an extent that depends on the operator length. In this paper, a new algorithm for an FX deconvolution filter is proposed based on a pseudo ARMA model. The deconvolution step in this algorithm can recover the smeared signal which is generated by a conventional FX filter.

### A brief outline of FX filter

With the assumption that each data point can be expressed as a linear combination of previous ones, the FX prediction filter for 2D data,  $x(t,n)$ , can be formulated as

$$X(f, n) = \sum_{k=1}^p c_k X(f, n - k) \quad , \quad n=1, 2 \dots N \quad (1)$$

Where  $X$  is a frequency slice with spatial sequential index  $n$ , and  $c$  is a filter operator with length  $p$ . The application of this filter includes two steps. First, find the filter  $c$  by solving equation (1) and secondly, apply the filter to each frequency slice. Applying this filter is based on the assumption that seismic events are linearly continuous and noise is random. However, in the real world, linearly continuous events may not exist and therefore, some smearing on the edges of events is inevitable as a result of the convolution.

As an alternative to the AR model, the FX ARMA method works on estimating noise and the filtered seismic data is the result of subtracting the estimated noise from the original data. This method can be equivalently formulated by assuming  $X = S + W$ , where  $S$  is pure signal and  $W$  is additive white noise, then (Sacchi and Kuehl, 2000)

$$\sum_{k=0}^p g_k X_{n-k} = \sum_{k=0}^p g_k W_{n-k} \quad (2)$$

where  $g$  is the prediction error filter with  $g(0)=1$ , and  $g(1:p)=-c(1:p)$ . Equation (2) requires that

$$\sum_{k=1}^p c_k S(f, n - k) = 0 \quad (3)$$

i.e. signal is linearly continuous and predicted error is purely white. Then from equation (2) we can estimate the white noise. Obviously, in normal seismic data, such a requirement cannot be satisfied. Moreover, if equation (3) is satisfied, white noise can be directly estimated by the prediction error filter and therefore, there is no need to solve equation (2). In such a situation, FX projection does not have any advantage over the normal FX filter. When equation (3) is not satisfied, i.e. signal cannot be well predicted, equation (2) cannot be used to correctly estimate white noise. Moreover, because the same operator  $g$  is used in both sides of equation (2),  $g$  is band limited and the estimated noise from equation (2) is a smooth version of the errors that are produced by the prediction error filter. As the result, the smeared edge cannot be recovered.

## Pseudo ARMA FX deconvolution

While using the same idea as that in the FX ARMA filter, Pseudo ARMA FX deconvolution is applied to the predicted signal, i.e.

$$\sum_{k=1}^p c_k X_{n-k} = \sum_{k=1}^p c_k \hat{X}_{n-k} \quad (4)$$

where  $\hat{X}$  is the final solution.

Solving equation (4) involves three stages: 1) determining the operator of the prediction filter, 2) apply the filter to obtain the FX result and 3) deconvolution of the FX result with the same operator of the prediction filter to obtain the final result. The whole procedure is called Pseudo ARMA FX because we do not really optimally estimate operators that are used for ARMA( $p,q$ ) but rather we apply the same prediction filters. Also, we solve equation (4) in separated stages.

It is important to point out that for pure signal data containing only linear events, (i.e. equation (3) is satisfied), our procedure should actually do nothing to the data. Otherwise, because of deconvolution, our procedure would simply reproduce the original data. Then, we must ask: how can it remove noise? If, in stage two, all the white noise has been totally removed, then the deconvolution stage should not produce white noise. The problem is that any portion of white noise left after stage two will be amplified during the deconvolution because the deconvolution problem is ill-posed. The key for success of our algorithm is that deconvolution is only applied to those portions of the data where we can identify signal with high reliability. Therefore, we apply the deconvolution in the Fourier-wavelet domain, i.e. *ForWaRD* method (Neelamani, et al, 2004), where signal and noise are well separated.

## Example

The first example is for synthetic data as shown in Figure 1. In the Figure, (a) is the input noisy data with a discontinuity in the linear events that models a fault, (b) is the output from conventional FX and it shows that the discontinuity is smeared, (c) is the output from Pseudo ARMA FX deconvolution and it shows that while white noise has been removed the sharp discontinuity also been preserved.

The second example uses real stacked data and is shown in Figure 2. In order to show the effect of our algorithm, the data has been shifted up on the right half portion of traces (a). Similarly to the example shown above, (b) is the output from conventional FX and it shows that the discontinuity is smeared. (c) is the output from Pseudo ARMA FX deconvolution and it shows that, while white noise has been removed, the details of the original data have also been preserved.

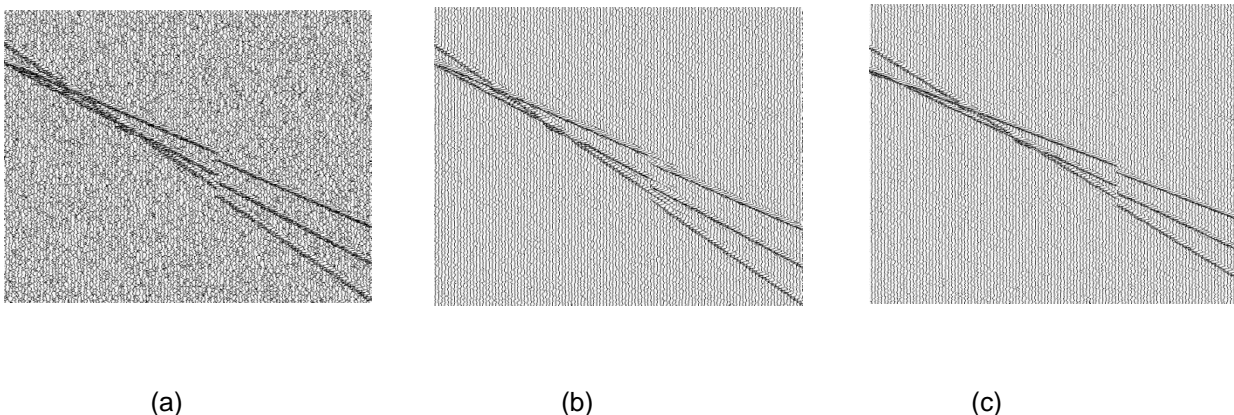


Figure 1. Synthetic data: (a) input; (b) result from FX and (c) result from Pseudo ARMA FX deconvolution

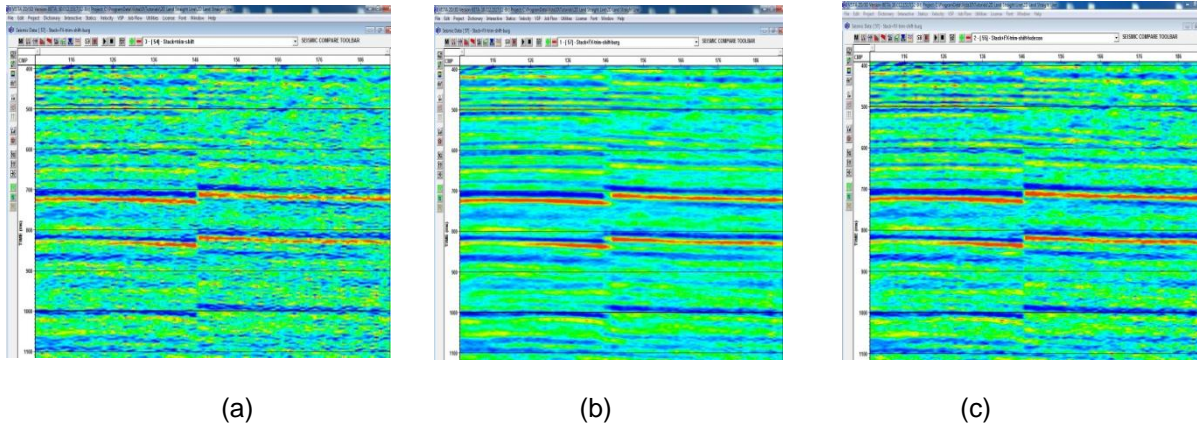


Figure 2. Real data: (a) input; (b) result from FX and (c) result from Pseudo ARMA FX deconvolution.

## Conclusions

We have presented the Pseudo ARMA FX deconvolution algorithm for seismic data noise reduction. This algorithm can be thought of as an extension of conventional FX with a deconvolution added. Therefore, while it does not suffer the drawback of FX smearing the edge boundaries, it does have the advantages of FX. The deconvolution should apply only to high amplitude reliable signal and while the wavelet domain is a good choice for deconvolution it is not the only one. The examples given above show that our algorithm works well for seismic data with geological faults.

## References

- Neelamani, Ramesh , Hyeokho Choi and Richard Baraniuk, ForWaRD: Fourier-Wavelet Regularized Deconvolution for Ill-Conditioned Systems, 2004, *IEEE Trans. on Signal processing*, Vol.52, No. 2.
- Sacchi, M.D and Henning Kuehl, 2000, FX ARMA Filters, *SEG Expanded Abstracts*.