A study of spectral broadening methods for multicomponent data

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Summary

Two methods have recently been published for carrying out nonstationary spectral broadening (and narrowing) of PS data after it has been mapped into the PP time domain. We present a study which investigates these two papers by Bansal & Matheney (2010) and by Gaiser (2011) (see also Gaiser et al., 2011a,b).

We find that the two approaches differ not only in their purposes, but also in their method of PS-to-PP time mapping, their proposed spectral corrections, and their methods for applying those corrections. Both papers make useful contributions to this field, and we try to add some clarification of fundamental resolution issues and illustrate our points with simple synthetic seismograms.

Introduction

A very common observation, and shortcoming, in converted-wave data (also called C-wave and/or PS data) is its poor resolution compared to PP data. Even after squeezing PS data from PS to PP time, the frequency content of the PS data is typically not as high as P-wave data (except for some important shallow reflector, near-surface exceptions). In the absence of attenuation due to Qp and Qs, we know that shear waves should provide better resolution than P-waves because the lower S-velocities have smaller spatial wavelengths (for the same temporal frequency) than the P-wavelengths. The fact that the highest frequencies in the PS data are not observed to be as high as in the PP data has typically been attributed to the fact that Qs has a more severe attenuative effect on the shear waves than the effect of Qp on the P-waves.

Recently Bansal & Matheney (2010) (hereafter referred to as Paper I) published a method for equalizing the wavelets in PS data after squeezing the time coordinate from PS time to PP time in order the prepare the data for inversion since a time-stationary wavelet is assumed to exist by inversion algorithms. Their method can generate some enhancement of the frequency content of the PS data after it is squeezed to PP time but this is not surprising since it involves a controlled form of time-varying spectral whitening which is an industry-standard method of trying to extract as much resolution out of data as possible by whitening its amplitude spectrum.

Even more recently Gaiser (2011) (hereafter referred to as Paper II) has published material where he suggests that we have all been underestimating the true resolving power of a lot of PS data. According to Gaiser, we have not been achieving the true resolution of the PS data after squeezing it to PP time (especially land PS data) because we have not been recognizing that the "wavelengths of P- and S-waves must match in order to sample reflectivity in an equivalent manner" (Gaiser, 2011). In contrast to Bansal and Matheney's conventional method of whitening existing temporal frequencies in the PS data, Gaiser's method involves an unorthodox mapping of the original frequencies in the data to higher, previously non-existent, frequencies. If Gaiser's point is correct, it would certainly be important since it would provide an instant method of getting better resolution from many PS datasets.

The potential improvement in resolution from Gaiser's method motivated us to examine Gaiser's argument in detail. The analysis presented here of some basic concepts of domain mapping, resolution, and wavelength preservation leads us to question some of Gaiser's statements and show that Bansal & Matheney's approach is basically sound.

We start our analysis by describing two methods of mapping data from PS time to PP time. One method depends on interval velocities and the other depends on average velocities. Confusion between Bansal and Matheney's method and Gaiser's method starts here because, contrary to what Gaiser (2011) states, the PS to PP time conversion can depend either on interval or average velocity ratio. We will find that doing the PS to PP time conversion with locally constant average velocity ratios causes confounding wavelet distortions that do not occur when doing the conversion with the true average velocity ratios or with interval velocity ratios.

Two methods of squeezing PS data from PS time to PP time

There are at least two methods that can be used to map PS data from PS time to PP time. One of them constructs the squeezed PS trace using the interval Vp/Vs ratio, γ_{int}, and the other constructs the squeezed PS trace using a Vp/Vs ratio that is averaged from the surface, γ_0. The interval Vp/Vs-based method uses the factor $2/(1+\gamma_{int})$ to do the mapping sample by sample within each constant interval. The average Vp/Vs-based method uses the factor $2/(1+\gamma_0)$ to do the mapping sample by sample down the entire trace. The factor $2/(1+\gamma)$ is the ratio of traveltimes, t_{PP}/t_{PS}, either across a region of constant γ_{int}, in the case of the interval Vp/Vs-based method, or averaged from the surface, in the case of the average Vp/Vs-based method.

In the case of the interval Vp/Vs-based method, let us assume that the γ_{int} model is blocky: i.e. Vp/Vs is constant within a block of time samples. So we start with a set of γ_{int} values that are used to map time samples within blocks of the original PS trace to blocks of samples in the squeezed PS trace in the following way. Beginning at the top, the squeezed PS trace is constructed interval by interval by first squeezing the sample interval, Δt_{PS}, of the top interval of the original PS trace by a constant factor, $2/(1+\gamma_{int})$, interpolating the squeezed samples within that interval to the desired Δt_{PP} of the output trace, and then placing those interpolated samples at the top of the squeezed PS trace. This procedure would be repeated for the samples within the second constant γ_{int} interval and those squeezed and interpolated samples would be pasted below the top layer of the squeezed PS trace. And then the third layer would be squeezed, interpolated and pasted below the second layer of samples, and so on to the bottom of the trace.

In the case of the average Vp/Vs-based squeezing method, we start with a γ_0 model which will typically vary sample by sample down the trace (except for the top layer if the underlying γ_{int} model is blocky). Therefore, the squeeze factor $2/(1+\gamma_0)$ defines a point-by-point mapping of samples in the original PS trace to samples in the squeezed PS trace. Since γ_0 typically varies sample by sample, then the amount of squeezing varies sample by sample as well. Interpolation will be required to do the mapping from constant Δt_{PS} samples in the original trace to constant Δt_{PP} samples in the squeezed PS trace.

Notice that the method of obtaining the γ_{int} or γ_0 models has not been described so far. It is important to distinguish the method of obtaining the Vp/Vs model from the method of performing PS time to PP time mapping. Gaiser (1996) describes a cross-correlation based method that naturally yields a γ_0 model, but he also describes a Dix-inversion method of converting the γ_0 values to a γ_{int} model. So Gaiser could use either method described above to do the PS time to PP time method, but his comments make it sound like he only uses the average Vp/Vs based method. Bansal & Matheney (2010) describe a method of registering PP-horizons to PS-horizons that naturally leads to a blocky γ_{int} model, and it sounds from their comments as if they have used this interval Vp/Vs model to do their PS time to PP mapping. But it would be a simple procedure to integrate Bansal & Matheney's γ_{int} model to obtain a γ_0

model and then do the mapping by the average Vp/Vs mapping method. The important point is that the method of mapping from PS to PP time does not need to be tied to the method of obtaining the Vp/Vs model.

The two mapping methods are compared in Figure 1, which shows a simple model and the associated γ_i and γ_0. Also shown are a PP trace and a PS trace compressed to PP time by each of the methods described above. It is clear that both methods result in identical wavelets that are compressed in time (higher frequency) relative to the PP wavelets, and that the arrival time of the events in PP time is correct. Both methods also result in nonstationary wavelets. However, it is important to note that the wavelets are stationary within a constant-γ_i interval, suggesting that their frequency bandwidth after compression is governed by γ_i rather than γ_0.

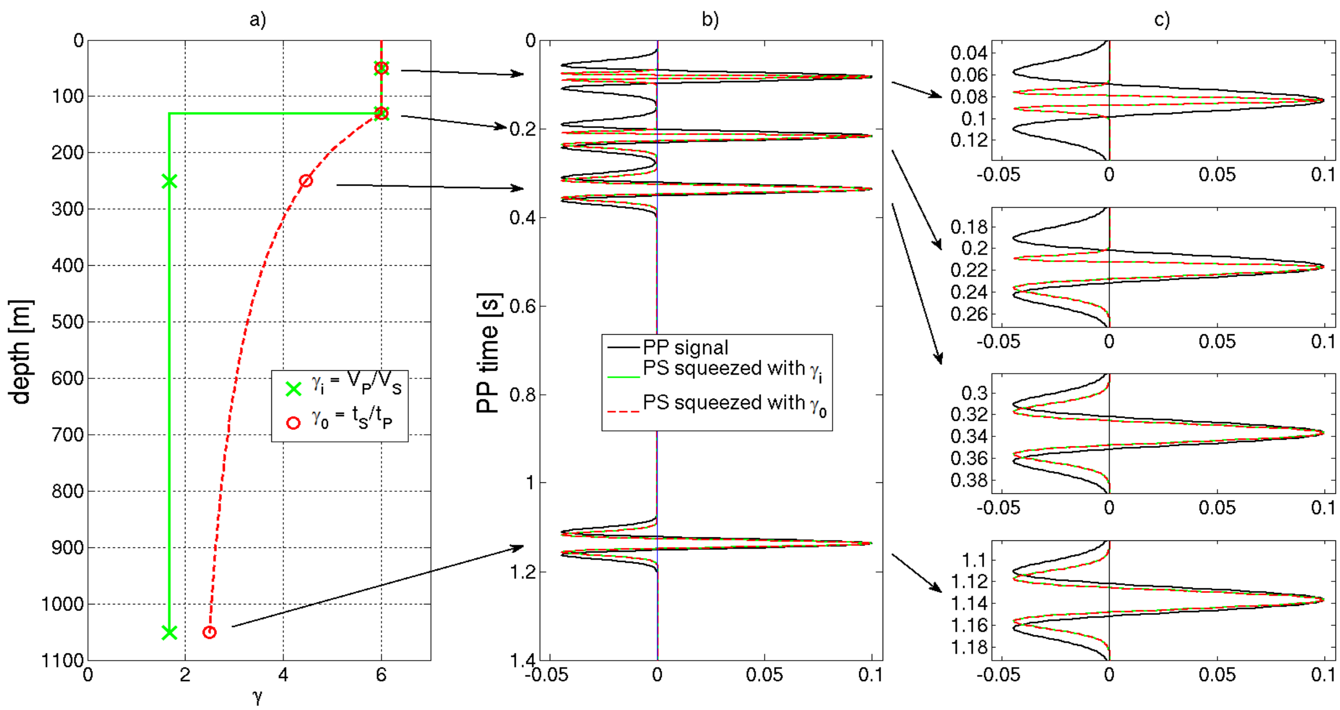

Figure 1: (a) Interval and average Vp/Vs values in depth for a simple model. The locations of four reflectors are indicated by symbols. (b) A PP trace (black line), a PS trace compressed to PP time by a method that uses interval V_P/V_S, γ_i (green line), and a PS trace compressed to PP time by a method that uses average V_P/V_S, γ_0 (dashed red line). (c) Close-ups of each of the four events in (b).

One point of particular interest is the asymmetric compressed wavelet of the second event. This is an artifact of the discontinuity in γ_i and in the derivative of γ_0, and should not be present. One possible approach is to modify γ so that it is constant in a region about each event. This yields well-behaved wavelets as shown in Figure 2.

However this procedure has an unintended consequence as well, namely that wavelets below the first layer possess a different frequency bandwidth when compressed by a locally constant γ_0, which we will denote γ_{0c}. In fact they are somewhat narrower than before which makes it seem that using γ_{0c} would be a useful way to enhance resolution. This is not the case, as we show in the next section, but first we will describe the wavelets in Figure 2 more precisely.

If the original wavelet (in either PP or PS traces) is characterized by a dominant frequency of f_0, then, as shown in Appendix A, the dominant frequency of the PS wavelet squeezed to PP time using γ_0 (or γ_i) is $2f_0/(1+\gamma_i)$, while that of the wavelet squeezed using γ_{0c} is $2f_0/(1+\gamma_0)$ or, for clarity, $2f_0/(1+\gamma_{0c})$.

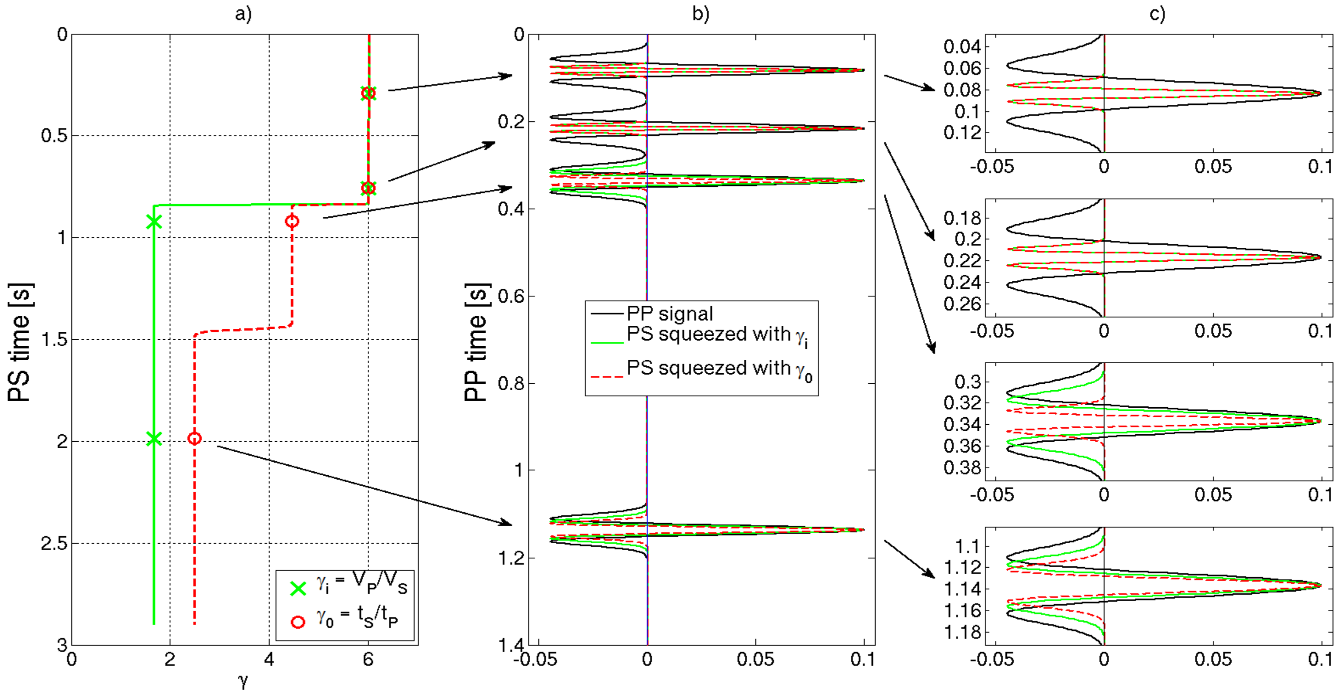

Figure 2: (a) Interval and average Vp/Vs values in PS for the same model as in Figure 1, but with values interpolated between event locations in such a way that Vp/Vs values are approximately constant in a region around each event. The PS times of four reflections are indicated by symbols. (b) A PP trace (black line), a PS trace compressed to PP time by a method that uses interval V_P/V_S, γ_i (green line), and a PS trace compressed to PP time by a method that uses locally constant average V_P/V_S, γ_{0c} (red line). (c) Close-ups of each of the four events in (b).

What is the importance of understanding how the PS-to-PP domain transformation is carried out? We address this question in our next section.

Wavelength = Resolution

In the last section we showed that the way in which frequency varies with time in squeezed PS traces depends upon the manner in which the domain transformation is carried out. This is important because frequency is related to wavelength, and wavelength is a fundamental measure of the resolving power of a wavefield. The P- and S-wavefield wavelengths in layer *i*, λ_{iP} and λ_{iS}, have a clear physical meaning. We show in Appendix B that a useful definition of λ_{iPS} for discussion of resolution is the harmonic average

$$1 / \lambda_{iPS} \equiv (1 / \lambda_{iP} + 1 / \lambda_{iS}) / 2. \tag{1}$$

The significance of this result is that, as discussed in Gaiser (1996), resolution is fundamentally related to wavelength. For instance if $V_{iP} / V_{iS} = 2$ for all layers then, before mapping from SS time to PP time, the SS signal will possess twice the resolving power of a PP signal with the same frequency bandwidth, because its wavelengths are half those of the PP wavefield. Thus if a PP signal can detect a layer of 20m thickness, then an SS signal can detect a layer of 10m thickness, and a PS signal a layer of ~13.3m thickness.

After mapping to PP time the SS signal would have an effective velocity of V_{iP} instead of V_{iS}, by virtue of it having been shifted in time, and the frequency of its time trace would now be 2 f_0. The doubling of the frequency does not mean however that its resolution has been doubled from what it was before the mapping, for the fact that its wavelength, $(2\ V_{iS}) / (2\ f_0) = \lambda_{iS}$, is unchanged means that its resolution has been *preserved* across the domain mapping. What the double frequency does tell us is that the resolution is still double that of the PP signal, for this can now be discerned from frequencies as well as wavelengths because the signals are now in the same time domain. The essential idea to take away from this is that resolution is fundamentally determined by wavelength, and this resolution can be preserved through various domain transformations, but it cannot be fundamentally increased once acquisition is complete.

One of the valuable points made in Paper II is that the frequency of domain-transformed PS wavelets should be corrected in such a way that wavelengths are correct. We would say that a wavelength is correct if it maintains the true resolving power of the original PS signal. After transforming to PP time the effective velocities of the PS signals become equal to P-wave velocities, and if the transformation employs γ_{0c}, then the frequency becomes f_0 $(1+\gamma_{0c})$ / 2. Thus the implied wavelength for a PS event from the bottom of interval i is $\lambda_{squeezed} = 2\ V_{iP}\ /\ [f_0\ (1+\gamma_{0c})\ ]$. As pointed out in Paper II, this is not the correct wavelength, and Paper II thus proposes a correction. We point out here though that if the transformation employs either γ_0 or γ_i then the frequency becomes f_0 $(1+\gamma_i)$ / 2. Thus the implied wavelength is $\lambda_{squeezed} = 2\ V_{iP}\ /\ [f_0\ (1+\gamma_i)\ ] = 2\ /\ [f_0\ (1\ /\ V_{iP} + 1\ /\ V_{iS})\ ]$, which is precisely the wavelength that correctly describes the PS signal's resolution.

In Figure 2 the result of squeezing with γ_{0c} produced narrower wavelets than for γ_0 or γ_i. This would perhaps make the use of γ_{0c} tempting in this case. To illustrate the danger of this though, we show in Figure 3 the effect of each transformation on tuned wavelets. Figure 3a is similar to the third event in Figure 2c, but the single event has been replaced by two closely spaced wavelets of opposite sign. This is a typical model for the important case of an embedded thin layer. In Figure 3b we show the γ_i -squeezed PS signal along with the "ideal" squeezed wavelet with the same frequency, i.e., the correct reflectivity has been convolved with a wavelet whose dominant frequency is $(1 + \gamma_i)/2$ times that of the original PS signal. We see that the two signals are identical, showing that the resolving power of the original PS signal has been preserved. In Figure 3c we show the γ_{0c}-squeezed PS signal along with its "ideal" squeezed wavelet i.e., the correct reflectivity convolved with a wavelet whose dominant frequency is $(1 + \gamma_0)/2$ times that of the original PS signal. Now we see that the two signals differ significantly and that the underlying reflectivity has been distorted by the squeezing process, which thus has not preserved the resolving power of the original PS signal.

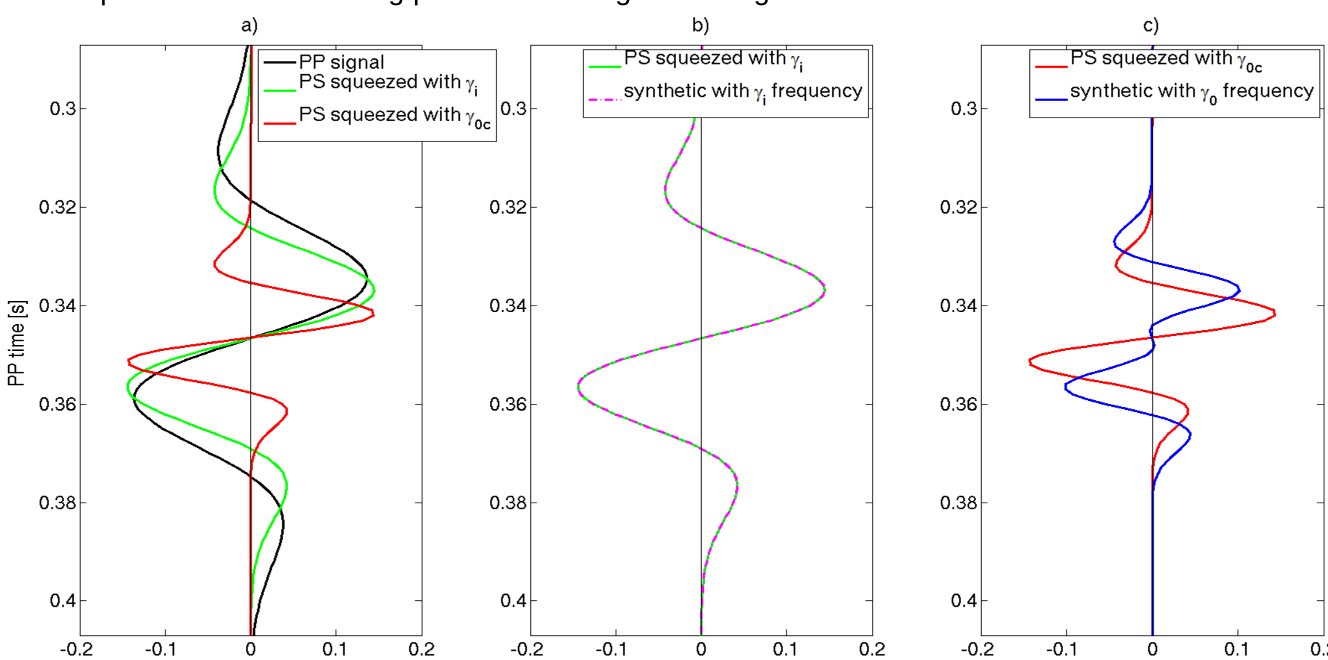

Figure 3: a) This panel is similar to the third event in Figure 2c, but the single event has been replaced by a pair of tuned events of opposite sign, located at 0.3367 s and 0.3567 s. b) The green line from Part a displayed together with a convolution of the reflectivity with a Ricker wavelet in which the dominant frequency has been multiplied by $2/(1+\gamma_i)$. Their exact coincidence shows that reflectivity has been preserved in squeezing with γ_i. c) The red line from Part a displayed together with a convolution of the reflectivity with a Ricker wavelet in which the dominant

frequency has been multiplied by $2/(1+\gamma_0)$. The difference between these lines shows that reflectivity has not been preserved in squeezing with γ_{0c}.

Thus a distinct advantage of performing PS-to-PP domain transforms with γ_i, as in Paper I, or with γ_0, is that the wavelengths automatically assume their correct values, as demonstrated in Figure 3. Then no correction is necessary.

Correction factors

There are of course situations when corrections are required. In the case of Paper I, it is necessary to obtain a stationary wavelet from the nonstationary squeezed wavelet. One might also wish to perform the PS-to-PP time conversion using γ_{0c} as part of the correlation method of Gaiser (1996). In this latter case, an important consequence of the analysis above is that to correct the wavelet one would need to apply the frequency scaling factor $(1 + \gamma_i) / (1 + \gamma_0)$. This differs from the suggested correction factor of Paper II, $2 \gamma_i / (1+ \gamma_0)$. This latter factor adjusts the frequency such that $\lambda_{iPS} = \lambda_{iS}$. While this may seem plausible in light of the upcoming PS signal being an S-wave, it nonetheless violates "conservation of resolution" by attempting to imbue the PS signal with the resolving power of an SS signal. The correction factor suggested here would restore the wavelength of a γ_{0c}-squeezed PS signal back to its original value.

Regardless of the differences between the various corrections described above, the one thing they all have in common is that they are nonstationary, and will thus require a nonstationary wavelet correction method. Paper I proposes a nonstationary filter (Margrave, 1998), while Paper II proposes a very different direct mapping, but discussion of these is beyond the scope of this paper.

Conclusions

We have demonstrated that wavelength and resolution are preserved for a PS signal compressed to PP time if the resulting wavelet has dominant frequency scaled by $(1+\gamma_i)/2$ relative to the frequency of the original wavelet. This condition is fulfilled for an underlying blocky model when PS-to-PP time mapping is carried out using interval Vp/Vs ratios or exact average Vp/Vs ratios. If one uses locally constant average Vp/Vs ratios then the frequency is scaled incorrectly and wavelengths are not preserved. Although the incorrect scaling may in some cases appear to increase resolution, we have shown that for tuned wavelets it can result in a distortion of the underlying reflectivity model.

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Appendix A: Bandwidths resulting from mapping with average Vp/Vs

Consider a PS trace wavelet centered at time t_0 and with zero-crossings at $t_0 - \Delta t$ and $t_0 + \Delta t$. (All times are in the PS domain unless indicated otherwise.) The time difference between these zero-crossings, $2\Delta t$, is in general inversely proportional to the bandwidth of the wavelet. What is the difference between compressing this wavelet to PP time with a locally constant average velocity ratio, γ_{0c}, or the true average velocity ratio, γ_0?

In the first case, because $\gamma_{0c}(t_0 \pm \Delta t) = \gamma_{0c}(t_0)$, the mapped time difference is given by

$$t_{PP}(t_0 + \Delta t) - t_{PP}(t_0 - \Delta t) = \frac{2(t_0 + \Delta t)}{1 + \gamma_{0c}(t_0 + \Delta t)} - \frac{2(t_0 - \Delta t)}{1 + \gamma_{0c}(t_0 - \Delta t)} = \frac{2}{1 + \gamma_{0c}(t_0)}(2\Delta t).$$

This shows that the bandwidth in the mapped wavelet will be scaled by $[1 + \gamma_{0c}(t_0)]/2 = [1 + \gamma_0(t_0)]/2$.

In the second case the mapped time difference is given by

$$t_{PP}(t_0 + \Delta t) - t_{PP}(t_0 - \Delta t) = \frac{2(t_0 + \Delta t)}{1 + \gamma_0(t_0 + \Delta t)} - \frac{2(t_0 - \Delta t)}{1 + \gamma_0(t_0 - \Delta t)}. \qquad \text{(A1)}$$

Now $\gamma_0(t_0) = t_{SS}(t_0)/t_{PP}(t_0)$ and if we define Δz as the depth difference associated with Δt, i.e.,

$$\Delta t = \frac{\Delta z}{V_{iP}} + \frac{\Delta z}{V_{iS}},$$

then we can write

$$\gamma_0(t_0 \pm \Delta t) = \frac{t_{SS}(t_0 \pm \Delta t)}{t_{PP}(t_0 \pm \Delta t)} = \frac{t_{SS}(t_0) \pm \frac{2\Delta z}{V_{iS}}}{t_{PP}(t_0) \pm \frac{2\Delta z}{V_{iP}}} = \frac{t_{SS}(t_0) \pm \frac{2\Delta t}{V_{iS}} \Big/ \left(\frac{1}{V_{iP}} + \frac{1}{V_{iS}}\right)}{t_{PP}(t_0) \pm \frac{2\Delta t}{V_{iP}} \Big/ \left(\frac{1}{V_{iP}} + \frac{1}{V_{iS}}\right)} = \frac{t_{SS}(t_0) \pm \frac{2\gamma_i \Delta t}{1 + \gamma_i}}{t_{PP}(t_0) \pm \frac{2\Delta t}{1 + \gamma_i}}$$

$$\approx \frac{t_{SS}(t_0)}{t_{PP}(t_0)}\left(1 \pm \frac{2\gamma_i}{1 + \gamma_i}\frac{\Delta t}{t_{SS}(t_0)}\right)\left(1 \mp \frac{2}{1 + \gamma_i}\frac{\Delta t}{t_{PP}(t_0)}\right) \approx \gamma_0(t_0) \pm \frac{t_{SS}(t_0)}{t_{PP}(t_0)}\frac{2\Delta t}{1 + \gamma_i}\left(\frac{\gamma_i}{t_{SS}(t_0)} - \frac{1}{t_{PP}(t_0)}\right)$$

$$= \gamma_0(t_0) \pm \frac{2\Delta t}{t_{PP}(t_0)}\frac{\gamma_i - \gamma_0(t_0)}{1 + \gamma_i}$$

Substituting this expression for $\gamma_0(t_0 \pm \Delta t)$ into Equation A1 yields

$$\frac{2(t_0 + \Delta t)}{1 + \gamma_0(t_0) + \frac{2\Delta t}{t_{PP}(t_0)}\frac{\gamma_i - \gamma_0(t_0)}{1 + \gamma_i}} - \frac{2(t_0 - \Delta t)}{1 + \gamma_0(t_0) - \frac{2\Delta t}{t_{PP}(t_0)}\frac{\gamma_i - \gamma_0(t_0)}{1 + \gamma_i}},$$

which, after expanding to linear order in $2\Delta t$, reduces to

$$t_{PP}(t_0 + \Delta t) - t_{PP}(t_0 - \Delta t) = \frac{2}{1 + \gamma_i}(2\Delta t) + O(\Delta t^2).$$

Thus using the true γ_0 to map from PS time to PP time gives the same mapping as mapping with γ_i, and results in a frequency bandwidth scaled by $(1 + \gamma_i)/2$.

Appendix B: Definition of effective wavelength for PS wavefield

Suppose that PP, PS, and SS signals with the same dominant frequency, f_0, are all received from a pair of reflectors separated by a distance *z*. It is straightforward to calculate the time difference between the two signals for each wavefield, namely

$$\Delta t_{PP} = 2z / V_{iP},$$
$$\Delta t_{PS} = z / V_{iP} + z / V_{iS},$$
$$\Delta t_{SS} = 2z / V_{iS}.$$

Now suppose that, for the given bandwidth, two signals in time can be distinguished if separated by time *T*. By setting $\Delta t_{ij} = T$ we can calculate the minimum *z* that can be distinguished by each wavefield, namely

$$z_{\mathrm{min,PP}} = T\, V_{iP} \;/\; 2,$$
$$z_{\mathrm{min,PS}} = T \,/\, (\, 1 / V_{iP} + 1 / V_{iS}\,),$$
$$z_{\mathrm{min,SS}} = T\, V_{iS} \;/\; 2.$$

We know that P- and S-wavelengths in this layer are $\lambda_{iP} = V_{iP} / f_0$ and $\lambda_{iS} = V_{iS} / f_0$, so we can also write

$$z_{\mathrm{min,PP}} = (T\, f_0 / 2)\, \lambda_{iP}, \quad \text{(B1)}$$
$$z_{\mathrm{min,SS}} = (T\, f_0 / 2)\, \lambda_{iS}. \quad \text{(B2)}$$

We see that in both B1 and B2 the minimum thickness is related to the wavelength by the same proportionality constant, $(T\, f_0 / 2)$. Various authors have defined this quantity in different ways, but typically it is in the range 0.1-0.3 (Meier & Lee, 2009). We are not concerned with its numerical value here, but if we wish to define an effective PS “wavelength” which contains information about the resolving power of a PS wave, we would use the same proportionality factor, $(T\, f_0 / 2)$, and write

$$1 / \lambda_{iPS} = (T\, f_0 / 2) / z_{\mathrm{min,PS}} = f_0\, (\, 1 / V_{iP} + 1 / V_{iS}\,) / 2 = (\, 1 / \lambda_{iP} + 1 / \lambda_{iS}\,) / 2 \quad \text{(B3)}$$

so that an effective λ_{iPS} is the harmonic average of P- and S-wavelengths, as stated in Equation 1.

This not intended to suggest that λ_{iPS} corresponds to an actual physical wavelength, only that it plays an analogous role to λ_{iP} and λ_{iS} in describing the resolving power of a PS signal.