Low-Frequency Electrical Conduction in Simple Porous Media

David C. Herrick Ph D, Yellowstone Petrophysics LLC, Cody, Wyoming, USA

davidcherrick@gmail.com

Summary

'Archie's law' has been the basis for virtually all saturation equations used in the petroleum industry for the past seventy years, however it has only an empirical foundation and its parameters are only qualitatively interpretable. A consideration of the basic physics of electrical conduction, however, leads to a simple, physically interpretable description of simple porous media of the type for which 'Archie's Law' was formulated.

Introduction

'Archie's law' is used throughout the petroleum industry as the basis of a quantitative method for estimating the hydrocarbon content of reservoirs. It is referenced and used in a great many published papers and books. Despite being regarded by many as a 'law' of nature, 'Archie's law' or 'Archie's equation' has only an empirical origin reflecting the curve fitting methods generally available in the early 1940s. Attempts have been made to derive 'Archie's law' from basic physical principles with little success. Rather than attempt to derive 'Archie's law', it is more fruitful to simply obtain a suitable description of the conductivity of porous materials using a simple analysis of the problem and the physics of conduction.

Conductivity of porous media

In the 1942 paper in which Archie gave the results of his empirical study, he defined the "formation resistivity factor" as the ratio of the resistivity of a brine-saturated rock to the resistivity of the saturating brine, R_0/R_w. By dividing out the brine resistivity, the formation resistivity factor F becomes a property solely of the brine-saturated rock, independent of the resistivity of the brine in the rock. Further consideration shows that there are only two additional properties of porous media that determine its conductivity: the pore volume and the pore geometry. One can carry Archie's approach one step beyond dividing out the effect of the brine conductivity by also dividing out the effect of brine volume leaving only the effect of the pore geometry. This pore-geometric effect can be referred to as the "geometric factor" symbolized as E_0. Effectively, the formation factor is separated into its two components, porosity and geometric factor. Separating the two components is important since, in general porous media, the two can be varied independently and many combinations of porosity and geometric factor give the same formation resistivity factor. This non-uniqueness of the formation resistivity factor makes interpretation of porous media data difficult and has resulted in much confusion in discussions and in the literature.

The separation of the formation resistivity factor into its component parts results in a simple description of the conductivity of brine-saturated porous media,

$$C_0 = C_w \, \phi \, E_0, \tag{1}$$

where C_0 is the conductivity of a brine-saturated porous medium, ϕ and E_0 are its porosity and geometric factor, respectively, and C_w is the brine conductivity.

A comparable equation for the conductivity of a partially brine saturated porous medium can be written by correcting the pore volume and pore geometry for the change in volume and geometry of the water in the medium due to its partial removal. The corrected brine volume is $S_w\phi$ and the corrected geometric factor is $e_t E_0$, where S_w is the fraction of the pore volume containing brine and e_t is the fractional change in E_0 resulting from the change in brine geometry due to desaturation. The conductivity equation for a partially brine-saturated porous medium C_t is

$$C_t = C_w \, S_w \phi \, e_t E_0. \tag{2}$$

$S_w\phi$ is the brine content of the partially brine-saturated medium and $e_t E_0$ is its geometric factor.

The pore-geometric factor E_0 can be understood explicitly by considering how electric currents flow through a porous medium. For simplicity of explanation I will consider a simple 2D porous medium, figure 1.
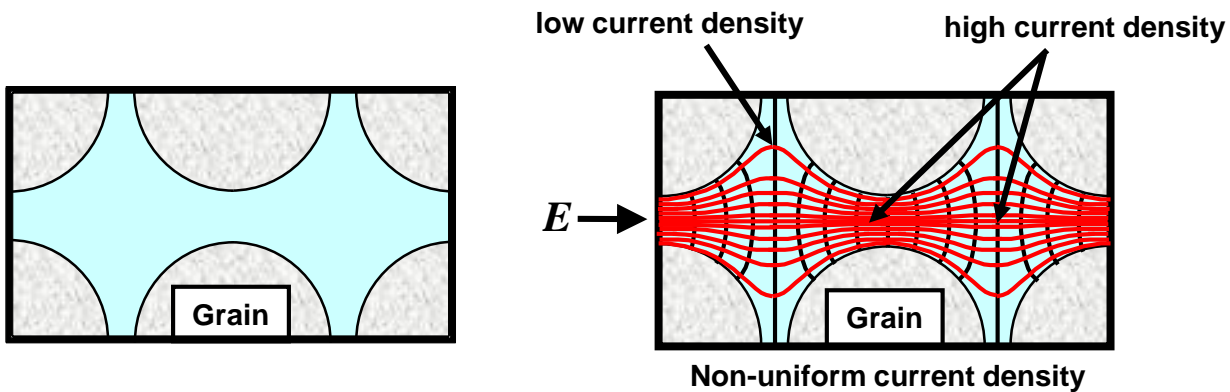

Figure 1. Simple brine-saturated 2D porous medium.

Figure 2. Same medium as in figure 1 in an electric field E. Vertically oriented black curves are equi-potential surfaces of the electric field gradient. Red curves are the current streamlines.

The circular grains in figure 1 are non-conductive; the saturating brine (blue) is highly conductive. Figure 2 shows the same medium in an electric field E oriented parallel to the horizontal axis. The electric field gradient can be determined by solving Laplace's equation. The solution for the pore geometry of figure 1 is shown in figure 2 as equi-potential surfaces which are depicted as black vertically oriented curves which can be thought of as electrical potential contour lines.

Current streamlines (red curves, figure 2) are always perpendicular to the equi-potential surfaces. If a uniform current is injected at the pore throat on the left, it flows down the potential gradient. As the current approaches the middle of the pore, it spreads out due to the curvature of the equi-potential surfaces. The result is a non-uniform current density distribution. In this illustration, the same amount of current is contained between each pair of streamlines. The current density is reflected in the spacing between streamlines. From the middle of the pore, the current density decreases vertically in both directions. The highest current density is in the pore throats and in the center on the pores between

pore throats. The total amount of current is determined by the cross-sectional area of the smallest (or limiting) pore throats.

The pore geometric factor E_0 can now be understood explicitly in terms of the non-uniform current density distribution observed in porous media and the cross-sectional area of the limiting constrictions between pores. Solving equation 1 for E_0 gives

$$E_0 = C_0 / (C_w \, \phi) \tag{3}$$

The numerator of equation 3 is simply the conductivity of the porous material. The denominator is the result of normalizing the porous medium conductivity to the brine conductivity (Archie's formation factor) and the porosity. The denominator is also has the form of the conductivity of a thick-walled tube with the same porosity and brine conductivity as the porous medium. Pictorially, equation 3 is
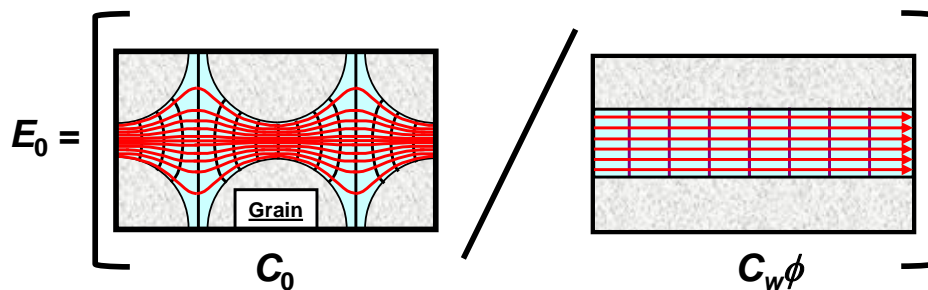

Figure 3. A pictorial representation of equation 3. E_0 is due to the non-uniform current density distribution and limiting constrictions resulting in conductivity C_0 as compared (by ratio) to an equivalent tube with the same dimensions, brine and porosity, $C_w \, \phi$

The denominator of equation 3 and of figure 3 is the porous material with the pore geometry removed leaving a straight thick-walled tube. Such a tube has the maximum conductivity for the combination of brine conductivity, porosity and external dimensions of the porous material. E_0 clearly gives the effect of the pore geometry of the porous medium on conductivity. The magnitude of E_0 is controlled by the current density distribution and the size of limiting constrictions.

E_0 can also be considered as the electrical efficiency of the pore system. The numerator of equation 3 expresses the degree to which the brine in the porous material conducts. The denominator expresses how well the brine could conduct if it had the tubular configuration for optimum conduction. For example a porous medium with $E_0 = 0.30$ means that the brine in the material conducts with only 30% efficiency of that which it is capable. This is comparable to stating that only about 30% of the brine in the pore space is effectively conducting electricity, 70% of the brine does not contribute significantly to the conductivity of the porous material. .

Conclusions

The complications that result from trying to interpret Archie's empirical equation can be avoided by reconsidering the problem of describing the conductivity of porous materials. Normalizing the conductivity of a porous material to both brine conductivity and porosity separates volumetric, geometric and compositional effects and allows one to investigate them separately. Archie's formation factor is ambiguous since it is the product of pore volumetric (porosity) and pore geometric (geometric factor) effects leading to difficulties in interpreting the parameters of Archie's equation. Separating the

volumetric and geometric conduction effects gives greater and more explicit interpretive capability. The pore-geometric factor E_0 can be explicitly understood in terms of the pore geometry determining the current density distribution in a given pore system and the constrictions that limit the total current.