Complex-beam migration: non-recursive and recursive implementations

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Summary

We have developed a complex-beam method for shot-domain prestack depth migration. The method is flexible with input geometry and accurate in imaging multipathing arrivals. It is especially useful for depth imaging of geologically complex land areas, where data-acquisition geometries are often irregular and sparse, and topographic variations are large. We show that the new method is a true-amplitude extension of the phase-shift wave-equation migration method into media with lateral velocity variations. It can be implemented both non-recursively and recursively. While the non-recursive implementation is efficient, the recursive implementation improves subsurface illumination, making the beam method comparable to reverse-time migration.

Introduction

Great progress has been made over the past few years in seismic imaging of marine data. Advanced methods such as reverse-time migration (RTM) have become standard tools for marine prestack depth migration. Yet, for land data, the imaging method of choice is still Kirchhoff migration (KM), generally the least accurate depth migration method because of its weakness in imaging multipathing arrivals. This is largely due to the fact that RTM is less flexible than KM in dealing with irregular, sparse data-acquisition geometries and large topographic variations – two problems often encountered in land surveys. Aiming at providing an advanced method for land imaging, we have developed a complex-beam method for shot-domain prestack depth migration. Based on a complex-beam solution to the wave equation, this method retains the flexibility of KM, but can also image multipathing arrivals. In this paper, we describe this method and show that the method is a true-amplitude extension of the phase-shift wave-equation migration (WEM) method. The method can be implemented both non-recursively and recursively. We also present migration results from a non-recursive implementation and demonstrate that the new method produces depth images superior to those from KM. It overcomes the dip limitation of WEM and compares well to RTM in imaging steep and overturned structures.

Complex-beam summation solution

A complex-beam summation solution to the wave equation, which forms the basis of complex-beam migration, has been derived by Zhu (2009). Here we summarize those results and establish notations used in this study. Consider the wave equation in the spatial domain $\mathbf{x} = (x, y, z)$

$$\nabla^2 u + \omega^2 v^{-2}(\mathbf{x}) u = 0, \tag{1}$$

where $u = u(\mathbf{x}, \omega)$ is the wavefield and $v(\mathbf{x})$ the velocity of the medium. The geometric-ray solution to this $\mathbf{x}$-domain wave equation takes the form

$$u(\mathbf{x}, \omega) = A(\mathbf{x}) e^{i\omega T(\mathbf{x})}, \tag{2}$$

where $A(\mathbf{x})$ and $T(\mathbf{x})$ are the amplitude and traveltime along the ray. The former is in turn given by

$$A(\mathbf{x}) = A(\mathbf{x}_0)\sqrt{\frac{v(\mathbf{x})J(\mathbf{x}_0)}{v(\mathbf{x}_0)J(\mathbf{x})}}, \tag{3}$$

where $\mathbf{x}_0$ is the source point and $J(\mathbf{x})$ the Jacobian of the transformation from the Cartesian to ray coordinates. Denote the slowness vector by $\mathbf{p} = (p_x, p_y, p_z)$ and the initial values of p_x and p_y at $\mathbf{x}_0$ by p_{0x}

and p_{0y}. The Maslov integral solution to wave equation 1 can then be written as (Chapman and Drummond, 1982):

$$u(\mathbf{x},\omega)=\iint U(\mathbf{y},\omega)e^{i\omega(p_x x+p_y y)}\frac{\partial(p_x,p_y)}{\partial(p_{0x},p_{0y})}dp_{0x}dp_{0y}, \quad (4)$$

where $U(\mathbf{y},\omega)$ is the Maslov solution to the wave equation in mixed domain $\mathbf{y}$ = (p_x, p_y, z) and given by

$$U(\mathbf{y},\omega)=B(\mathbf{y})e^{i\omega\tau(\mathbf{y})}. \quad (5)$$

Its amplitude $B(\mathbf{y})$ is calculated from the Jacobian along the geometric ray by

$$B(\mathbf{y})=B(\mathbf{y}_0)\sqrt{\frac{v(\mathbf{x})J(\mathbf{x}_0)\cos\theta}{v(\mathbf{x}_0)J(\mathbf{x})}\frac{\partial(x,y)}{\partial(p_x,p_y)}}=B(\mathbf{y}_0)G(\mathbf{y}_0,\mathbf{y}), \quad (6)$$

where θ is the take-off angle of the ray and $G(\mathbf{y}_0,\mathbf{y})$ denotes the geometric spreading in the **y** domain. The phase function $\tau(\mathbf{y})$ is given by the Legendre transform of the geometric-ray traveltime $T(\mathbf{x})$:

$$\tau(\mathbf{y})=T(x(\mathbf{y}),y(\mathbf{y}),z)-p_x x(\mathbf{y})-p_y y(\mathbf{y}). \quad (7)$$

Substituting equation 6 into equation 5 yields

$$B(\mathbf{y}_0)=U_0=U(p_x,p_y,0,\omega), \quad (8)$$

where $U(p_x,p_y 0,\omega)$ is given by the slant stack of the recorded wavefield $u(x,y,0,\omega)$:

$$U(p_x,p_y,0,\omega)=\left(\frac{\omega}{2\pi}\right)^2\iint u(x,y,0,\omega)e^{-i\omega(p_x x+p_y y)}dxdy. \quad (9)$$

Integral 4 represents a summation of geometric-ray solutions. It can be transformed into a summation of complex beams by replacing the geometric-ray solutions with complex-ray solutions (Zhu 2009). This gives

$$u(\mathbf{x},\omega)=\iint U_0 u_{CB}(\mathbf{x},\omega)dp_{0x}dp_{0y}, \quad (10)$$

where u_{CB} is a complex-beam solution constructed along a given geometric ray, often referred to as the central ray, in a ray-centered coordinate system. Details of this solution can be found in Zhu (2009).

It can be shown that in a homogeneous medium, $G(\mathbf{y}_0,\mathbf{y})\equiv 1$, and, with $p_x=k_x/\omega$ and $p_y=k_y/\omega$, the **y**-domain Maslov solution 5 reduces to the phase-shift solution

$$U(k_x,k_y,z,\omega)=U(k_x,k_y,0,\omega)e^{i\sqrt{\omega^2/v^2-k_x^2-k_y^2}z}, \quad (11)$$

which is the basis of the phase-shift WEM method. Since solution 5 includes geometric spreading $G(\mathbf{y}_0,\mathbf{y})$, summation 4, and hence 10, represents a true-amplitude extension of the phase-shift method into laterally inhomogeneous media. Similar to the phase-shift method, equation 10 can also be implemented recursively.

Non-recursive and recursive shot-domain complex-beam migration

Complex-beam migration based on extrapolation equation 10 is flexible in implementation: It can be implemented in the shot domain both non-recursively and recursively. Both implementations require local plane-wave decomposition of shot records, but differ in how the local planes are back extrapolated into the subsurface and imaged.

The local plane-wave decomposition of a shot record is accomplished by partitioning the record into small patches using overlapping Gaussian windows, each window centered on a regularly spaced grid point, referred to as a beam center. Slant stack 9 is then used to decompose each of these windowed data patches into local plane waves with a range of initial propagation directions. The number of initial directions is determined by wavefield sampling theory and the spacing between the beam centers by the partition of unity of the overlapping Gaussian windows (Hill, 2001).

For non-recursive implementation, each local plane wave from a beam center is back propagated by a single complex beam traced from the center with the plane-wave's initial direction across the entire migration aperture. Similar to the wavefield at the receivers, the point-source wavefield at the shot location is also decomposed into local plane waves and propagated by complex beams with different initial propagation directions. A subsurface image is generated from the overlapping volume between a pair of source and receiver beams. Accumulating the contributions from all shot-receiver beam pairs for a given beam centre produces a beam-center image, and summing over all beam centers from a shot record gives a common-shot image. The final subsurface image is formed by stacking together all individual common-shot images.

The non-recursive implementation is efficient but may suffer from poor subsurface illumination beneath a highly complex overburden, resulting in images inferior to those from RTM and WEM. It is common to attribute this inferiority to the high-frequency approximation of ray and beam methods. This is in fact a misconception as it can be shown that the P and S-wave decoupled acoustic wave equation used in RTM and WEM is of the same-order approximation as the ray and beam methods to the elastic wave equation in general inhomogeneous earth media (e.g., Ben-Menahem and Beydoun, 1985; Zhu, 1988). Both approximations assume local homogeneity of a medium, i.e., the variations of elastic parameters and velocities of the medium over a wavelength are much smaller compared to the parameters and velocities themselves. Moreover, as shown in the previous section, solution 5 is a true-amplitude extension of the phase-shift solution and the beam method is therefore at least as accurate as the phase-shift WEM method. Thus the inferiority is not due to the accuracy of the beam approximation. It is due to the implementation: RTM and WEM are implemented recursively and there are many propagation paths from each surface location illuminating a depth point (Gray and May, 1994), resulting a uniform spatial sampling of the subsurface by the wavefield. For the non-recursive implementation of the beam method, on the other hand, severe shadow zones can develop over long propagation paths in a complex medium so that there will be no propagation paths from a surface location to the depth points in some regions, resulting in poor illumination in these regions. This problem can be resolved by implementing the beam method recursively in a manner similar to that of the phase-shift WEM method. Such implementation regularizes the wavefield at each recursive depth, making the beam illumination comparable to those of RTM and WEM.

Example

We have completed the non-recursive implementation of the complex-beam method. The migration results from this implementation are compared here with those generated by Kirchhoff, WEM and RTM using a synthetic dataset from the 2D Canadian Foothills model shown in Figure 1a. The model is about 25km long and 10km deep and is characterized by large lateral variations in both velocity and topography. The velocity ranges from 3600 to 6000 m/s and the maximum elevation difference reaches 1340m. It also contains steep and overturned folds as highlighted by the ellipses in the figure. Based on finite-difference solution of wave equation 1, RTM is the most accurate migration method and its image in Figure 1b is used here as a benchmark for the comparison. For the most part of the model, all four methods give similar results. They all image, for example, the dipping basal reflector at the bottom of the model and the faulted fold about the reflector accurately. The Kirchhoff result (Figure 1c), however, shows more migration swing artifacts, in part because of its wide-spread migration operators. It also failed to image the steep and overturned structures. The WEM image (Figure 1d) appears to be cleaner than the other three, but WEM was also unable to image the steep structures due to its one-way approximation. Complex-beam migration (Figure 1e), on the other hand, compares well to the RTM method and imaged both the steep and overturned structures correctly.

Conclusions

Aiming at providing an advanced method for land imaging, we have developed a complex-beam method for shot-domain prestack depth migration. Based on a complex-beam summation solution, this method is flexible with input geometry and accurate in imaging multipathing arrivals. The beam method has the same order of accuracy as RTM in approximating the elastic wave equation in earth media and can be implemented both non-recursively and recursively. Test results from our non-recursive implementation demonstrate that the complex-beam images are superior to those produced by the Kirchhoff method. Moreover, the new method does not suffer the dip limitations of WEM and compares well to RTM in imaging steep to overturned structures. Quality of beam images, especially those beneath highly complex overburdens, can be further improved by a recursive implementation, which regularizes the subsurface illumination of the beams, making the method comparable to RTM.

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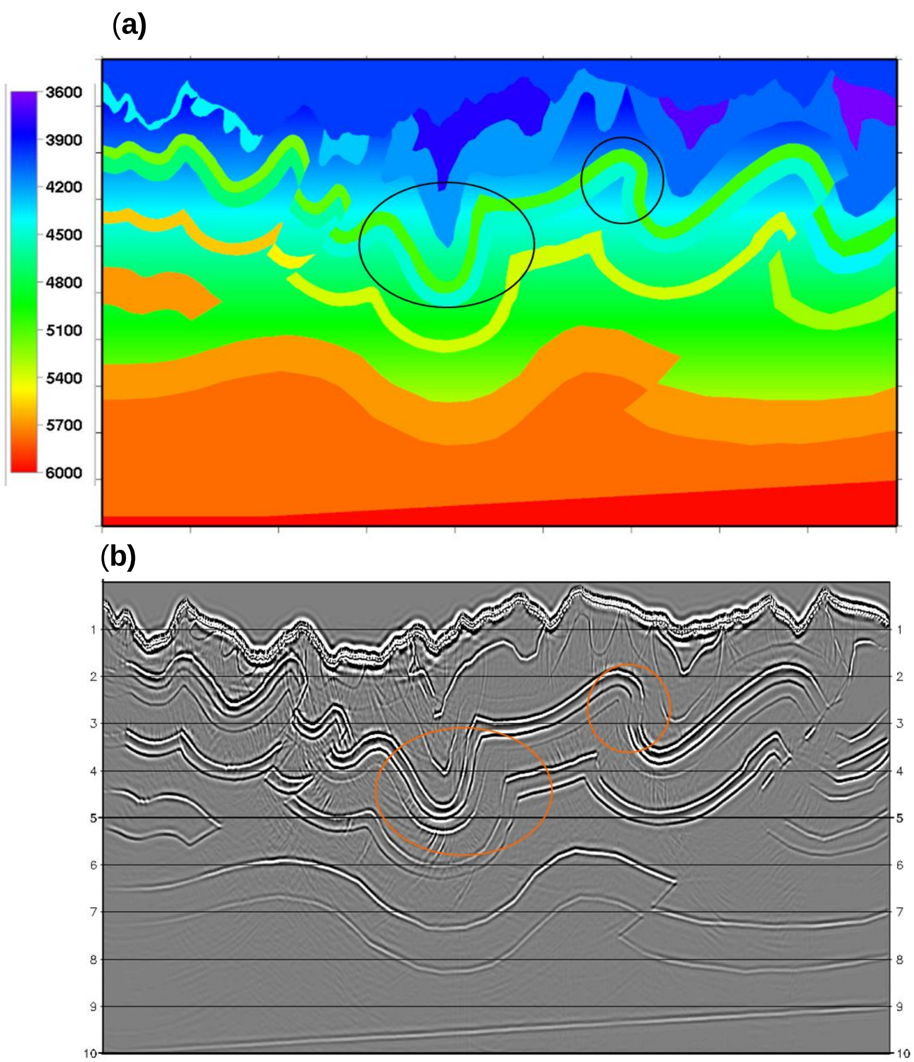

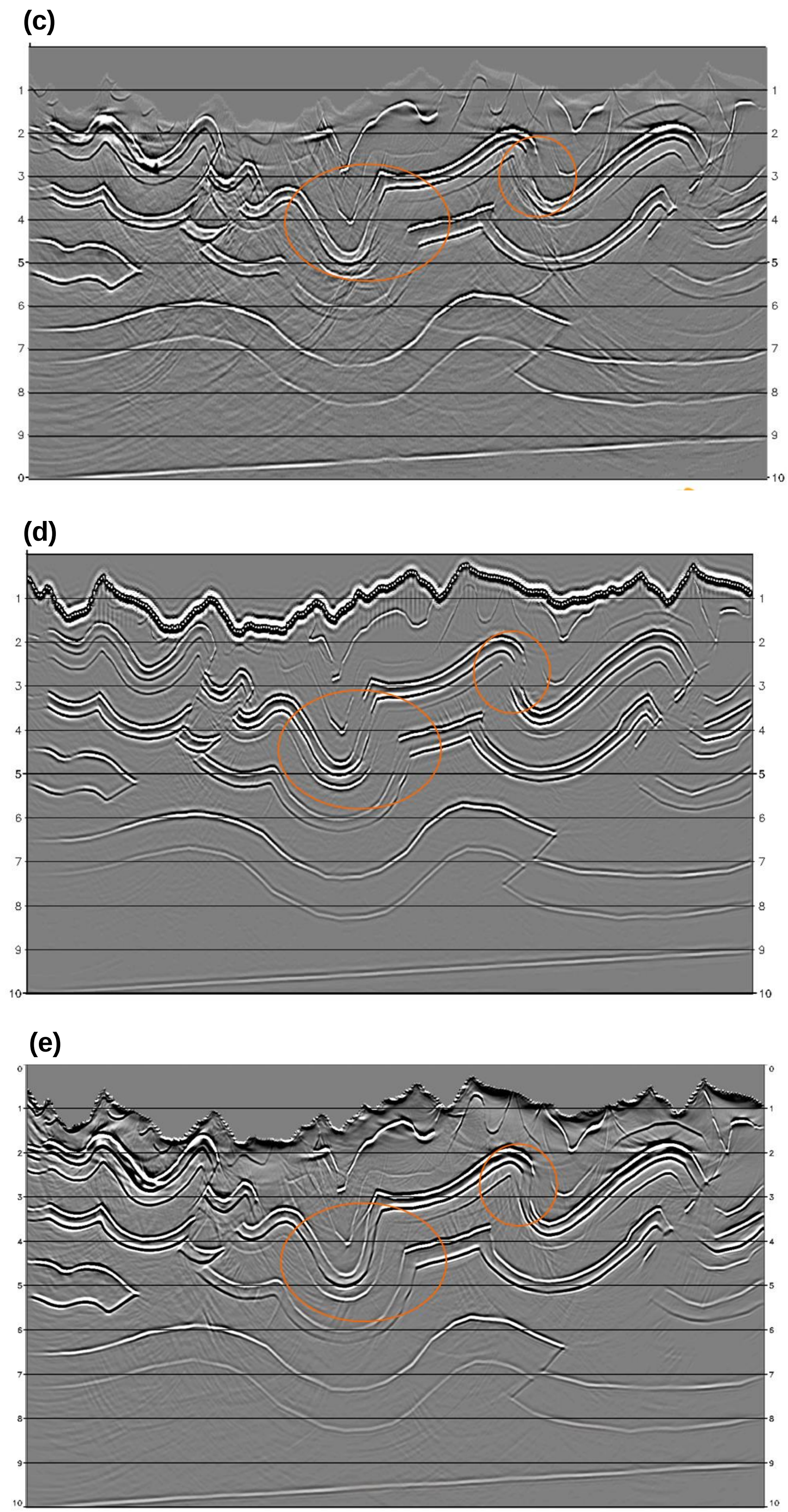

Figure 1. Depth migrated images from the Foothills model dataset. (a) Velocity model using for generating the synthetic data. (b) Reverse-time migration. (c) Kirchhoff migration. (d) Wave-equation migration. (e) Complex-beam migration.