Integration geoConvention 2013 | geoscience engineering partnership

Modified Euclid's blind deconvolution via sparsity optimization on a sphere

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Summary

We introduce a Sparse Multichannel Blind Deconvolution (SMBD) method. The method is a modification of Euclid's blind deconvolution where the multichannel impulse response is estimated by solving a homogeneous system of equations. SMBD can tolerate moderate levels of noise and does not require knowing the source duration. SMBD solves the homogeneous system of equations arising in Euclid deconvolution by imposing sparsity on the multichannel impulse response. To avoid the trivial solution, the sparse reflectivity is constrained to have unit norm. The latter results in non- convex optimization that is solved via a constrained steepest descent technique.

Introduction

Deconvolution is an important and recurrent topic in seismic data processing. Many signals and images can be represented via the convolution of an unknown signal of interest and a source signature. In general, the unknown signal or image can be estimated via inverse methods when the source signature is known. This process is called deconvolution. When the wavelet is unknown the process requires the simultaneous estimation of two signals. This process is often called blind deconvolution Shalvi and Weinstein (1990). Two early attempts on blind deconvolution are Homomorphic Deconvolution based on the work by Oppenheim and Schafer (1968) and implemented for the first time in exploration seismology by Ulrych (1971). Another technique for blind deconvolution was proposed by Wiggins (1985) who coined the name Minimum Entropy Deconvolution (MED) algorithm. MED assumes that the reflectivity is sparse and operates by finding an inverse filter that maximizes a measure of sparsity Donoho (1981). Both Homomorphic Deconvolution and MED suffer from a variety of shortcomings. For instance, homomorphic deconvolution is inclined to instability due to phase unwrapping and by its inherent inability to incorporate an additive noise term in the data. MED deconvolution often tends to annihilate small reflection coefficients Ooe and Ulrych (1979); Walden (1985).

Euclid deconvolution is a member of the plethora of methods that have been proposed for blind deconvolution of seismic data. The method was first discussed in the geophysical literature by Rietsch (1997a) and tested with real data examples in Rietsch (1997b). The method has also been investigated by Xu et al. (1995) and Liu and Malvar (2001) in communication theory and deconvolution of reverberations. The idea can be summarized as finding common factors of the z-transform of the source embedded in a group of seismograms with different reflectivity sequences. The problem leads to the estimation of the multichannel seismic reflectivity via the solution of homogeneous system of equations Mazzucchell and Spagnolini (2001). In the ideal case, the eigenvector associated to the minimum non-zero eigenvalue of the homogenous system of equations is an estimator of the multichannel reflectivity. However, small amounts of noise impinge on the identification of the eigenvector associated to the impulse response. We propose an improvement to Euclid deconvolution where the homogeneous equation is satisfied by a sparse solution (sparse impulse responses). In other words, we are assuming that the multichannel impulse response of the earth is a multichannel sparse series.

Theory

The earth can be modeled as a linear time invariant (LTI) system. The input-output relationship for this system, assuming a stationary source wavelet and a noise free condition, can be written as

$$d_j[n] = \sum_k w[n-k] r_j[k]\,, \quad j = 1 \dots J \tag{1}$$

where the multichannel seismic data can be represented by the $\mathbf{d}_j = (d_j[0], d_j[2], \dots, d_j[N-1])^T$, similarly the impulse responses for channel j can be written as $\mathbf{r}_j = (r_j[0], r_j[2], \dots, r_j[M-1])^T$, and finally we represent the seismic source function (the wavelet) via the vector $\mathbf{w} = (w[0], w[2], \dots, w[L-1])^T$. We stress that $N = M + L - 1$. We also remind the readers that convolution can be expressed using the z-transform as follows

$$D_j(z) = W(z) R_j(z)\,, \quad j = 1 \dots J\,. \tag{2}$$

By virtue of equation 2, it is easy to show that

$$D_p(z) R_q(z) - D_q(z) R_p(z) = 0\,, \quad \forall\, p, q \tag{3}$$

which can be rewritten in matrix-vector form as follows

$$\mathbf{D}_p\, \mathbf{r}_q - \mathbf{D}_q\, \mathbf{r}_p = \mathbf{0} \tag{4}$$

where $\mathbf{D}_p$ and $\mathbf{D}_q$ in equation 4 are the convolution matrices of channels p and q, respectively. Combining all possible equations of type 4 leads to the following homogeneous system of equations

$$\mathbf{A}\,\mathbf{x} = \mathbf{0} \tag{5}$$

where

$$\mathbf{A} = \begin{pmatrix} D_2 & -D_1 & & & & & \\ D_3 & & -D_1 & & & & \\ D_4 & & & -D_1 & & & \\ \vdots & & & & \ddots & & \\ & D_3 & -D_2 & & & & \\ & D_4 & & -D_2 & & & \\ & \vdots & & & \ddots & & \\ & & & & D_J & & -D_{J-2} \\ & & & & & D_J & -D_{J-1} \end{pmatrix} \tag{6}$$

and

$$\mathbf{x} = [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_J]^T\,. \tag{7}$$

Euclid method estimates the reflectivity by estimating the eigenvector associated to the minimum non-zero eigenvalue of $\mathbf{A}^T\mathbf{A}$ Rietsch (1997a). A small amount of noise in the data makes the solution impractical for real data applications Rietsch (1997b). The addition of a noise term in our signal model leads to

$$\mathbf{A}\mathbf{x} = \mathbf{e}\,. \tag{8}$$

It can be shown that $\mathbf{e}$ is white and we propose to find a solution $\mathbf{x}$ that minimizes $\mathbf{e}$. In addition we will constraint $\mathbf{x}$ to be sparse. To avoid the trivial solution, we must equip our problem with an extra constraint $\mathbf{x}^T\mathbf{x} = 1$. We propose to find the solution by minimizing the cost $J(\mathbf{x})$

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}}\, J(\mathbf{x}), \quad \text{subject to } \mathbf{x}^T\mathbf{x} = 1 \tag{9}$$

where

$$J(\mathbf{x}) = \frac{1}{2} ||\mathbf{A}\,\mathbf{x}||_2^2 + \lambda\, \mathscr{H}_\mu(\mathbf{x})\,. \tag{10}$$

The symbol $\mathscr{H}_\mu(.)$ is used to indicate the Huber norm with Huber parameter μ. The Huber norm forces solutions that are sparse. Using the Huber norm enables us to use a simple optimization method based on steepest descent techniques. It is worth mentioning that to preserve the constraint and keep the solution on the unit sphere, one should use an educated step that can be derived from Rodrigues' rotation formula Murray et al. (1994).

Examples

To examine the performance of proposed method we applied SMBD to a near offset section from the Gulf of Mexico seismic data (Figure 1a). Figures 1a and b show the data before and after deconvolution via SMBD, respectively. We also estimated the wavelet using the frequency-domain least squares estimator. The wavelet is portrayed in Figure 2b. For completeness, the average sea floor first break source wavelet is extracted and compared with the estimated wavelet. There is a strong resemblance of the estimated wavelet with the average first break pulse that was extracted by flattening and averaging the water bottom reflection.

Conclusion

We have presented a modification to Euclid's blind deconvolution method. The proposed method alleviates some of the problems encountered in blind deconvolution via Euclid method. For instance, SMBD can tolerate moderate levels of noise and does not require to know the precise length of the source duration. The method permits to estimate broad-band (sparse) impulse responses that can be used for wavelet estimation.

Acknowledgements

The authors are grateful to the sponsors of Signal Analysis and Imaging Group (SAIG) at the University of Alberta.

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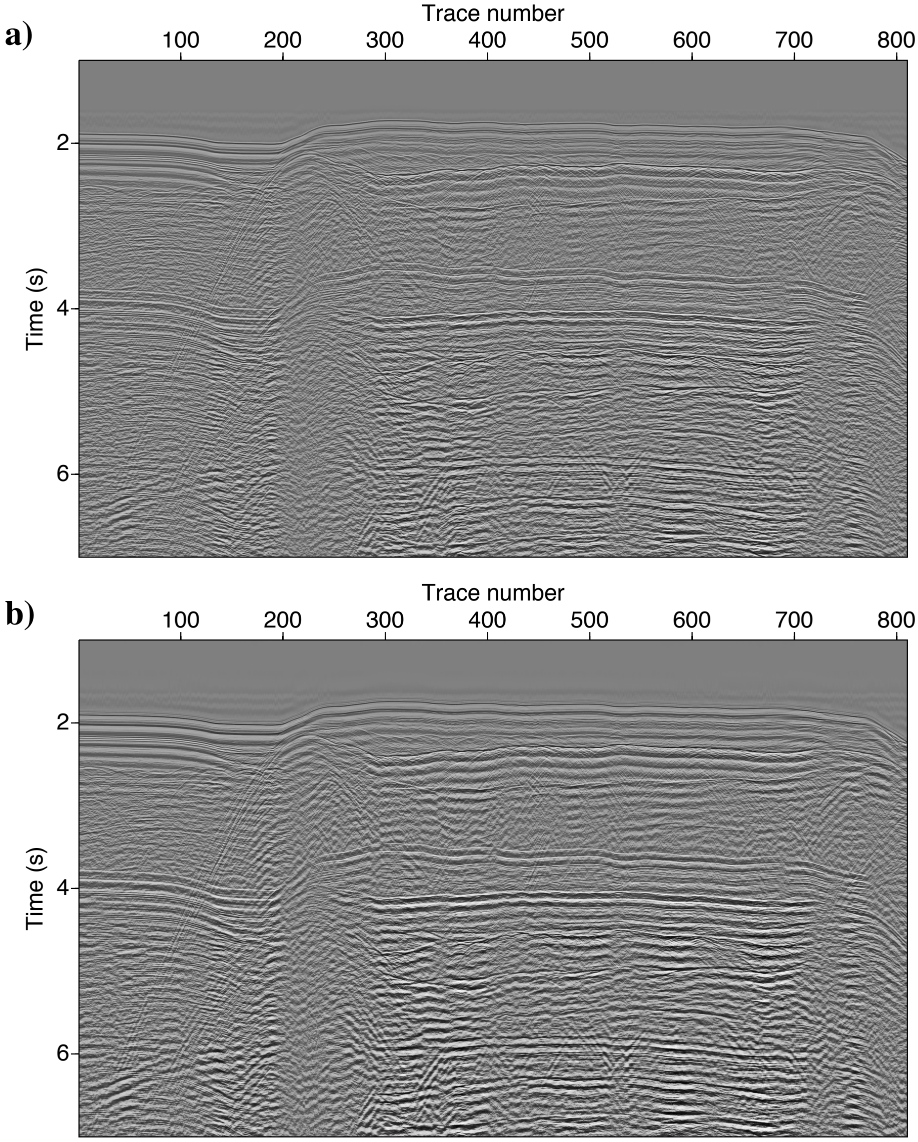

Figure 1 *Deconvolution of the Gulf of Mexico near offset section with the estimated wavelet. a) Before deconvolution. b) After deconvolution.*

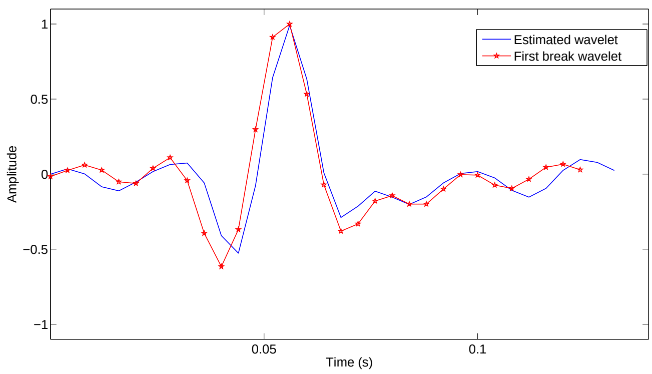

Figure 2 *:Comparison of extracted wavelet for Gulf of Mexico seismic data using SMBD method with first break pulse.*