Characterizing dipping fractures from P-wave Amplitude versus Azimuth studies

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Summary

This paper investigates the influence of fracture dip on P-wave Amplitude versus Azimuth (AVAz) reflectivity. In particular I derive a linearized equation describing how the P-wave reflectivity varies as a function of angle of incidence and azimuth at an interface composed of two anisotropic media. This derivation assumes that the anisotropy is the result of a single set of dipping fractures. Linear slip deformation (LSD) theory is used to model the anisotropy due to asymmetric fractures. It is assumed that the orientation of the fractures is the same in the two media but that the other fracture parameters are free to vary as a function of layer. In the subcase of rotationally invariant fractures this results in TTI media.

The linearized expression is written in terms of six independent azimuthal basis functions. The azimuthal reflectivity is a linear sum of these basis functions. To a first order, the dip influences the azimuthal functions by scaling them by the square of the cosine of the dip. The basis functions are nonlinearly dependent on the seven parameters describing the problem; the P-wave and S-wave background velocity reflectivity, the density reflectivity, the dip, and the three fractional fracture weakness parameters. The azimuthal basis functions represent an alternative data space. There are seven unknown parameters for six data basis functions so the inverse problem is fundamentally undetermined. This is consistent in the limiting case of horizontal fractures which results in a VTI media which is an underdetermined problem. One way to make the problem well posed is to add extra constraints to the problem such as specifying the fractures are vertical (HTI). The consequences of this assumption are explored by performing a modeling study. It is shown that significant biases are introduced into the estimates for dips greater than 30 degrees from vertical.

Introduction

The characterization of fractures using P-wave AVAz typically assume vertical fractures (Rüger 2002; Downton, 2011), but in reality most fractures do not meet this ideal abstraction. This paper investigates the amount of bias this assumption introduces. To achieve this goal, this paper generalizes the azimuthal Fourier Coefficient (FC) approach of Downton et al. (2011) to model dipping asymmetric fractures obtaining a linearized expression as a function of dip, fracture weakness parameters and the background isotropic parameters. The linearization is essentially an extension of the classic 3-term AVO expression (Swan, 1993) where the azimuthal reflectivity dependence is a function of three fracture weakness parameters and dip. Having established this relationship it is possible to investigate the effect of dip. I then show that the inverse problem is underdetermined hence necessitating making some *a priori* assumption such as the vertical fracture assumption. A modeling study is then performed to characterize the bias that this assumption introduces. In the modeling study significant bias occurs in the estimates when the fractures deviate greater than 30 degrees from vertical.

This derivation assumes that the anisotropy is the result of a single set of dipping fractures. In this paper the fracture dip is defined as the angular counterclockwise deviation of the normal of the fracture from horizontal. Essentially, this represents an angular deviation from vertical. Linear slip theory (Schoenberg, 1980) is used to model the anisotropy due to the fractures. It is assumed that the orientation of the fractures is the same in the two media but that the other fracture parameters are free to vary as a function of layer. The paper begins with a review of linear slip theory describing the elastic stiffness matrix of a single vertical fracture in an otherwise isotropic background media. A Bond transformation is then introduced so as to allow the fracture to dip.

Having calculated the stiffness matrix for a dipping fracture, the reflectivity expressions of Downton et al. (2011) are used to calculate the AVAz reflectivity for a dipping fracture with arbitrary strike. I show that the 2nd basis function coefficient w_{12} is to a linear approximation equivalent to the anisotropic gradient parameter B_{ani} estimated by Rüger. By showing how dip influences w_{12}, it is possible to predict how the B_{ani} estimate will be biased. Having established the near offset behavior the far offset behavior is also discussed.

Lastly, the uniqueness of the inverse problem is discussed. The forward problem is described in a minimal fashion using the azimuthal basis reflectivity expression. By writing the forward problem in this form, it is possible to show the inverse problem is underdetermined since the number of unknown parameters is greater than the number of data. In order to solve the problem uniquely some assumption must be made, two possible assumptions being that the fractures are vertical or they are rotationally invariant. The paper concludes by showing the biases introduced into the estimates by making the assumption that the fractures are vertical when in actual fact they have dip.

Linear slip theory

The LSD theory (Schoenberg, 1980) allows fractures to be modeled as a perturbation of the compliance of the background rock. The total compliance of the rock **S** is the sum of the background compliance $\mathbf{S}_b$ plus the compliance due to the fractures $\mathbf{S}_f$. The fractures can be modeled as an imperfectly bonded interface where the traction is continuous but the displacement might be discontinuous. The displacement discontinuity is linearly related to the traction. For example, the displacement discontinuity normal to the fracture is proportional to the normal stress. This proportionality constant is the normal fracture compliance B_N. Similarly, the vertical and horizontal fracture compliances B_V and B_H may be defined. In the case that both the vertical and horizontal fracture compliances are the same the fracture is said to be rotationally invariant. This is the case of penny shaped fractures and for vertical fractures normal to the x-axis gives rise to HTI anisotropy (Schoenberg and Sayers, 1995). When B_V and B_H are different the fracture is said to be asymmetric. This is probably the more realistic case (Far, 2011) but requires more parameters introducing extra complexity. Asymmetric fractures give rise to orthorhombic anisotropy.

Instead of working with compliances, I choose to parameterize the problem in terms of the normal, vertical and horizontal fracture weakness parameters

$$\delta_N = \frac{MB_N}{1+MB_N}, \tag{1}$$

$$\delta_V = \frac{\mu B_V}{1+\mu B_V}, \tag{2}$$

and

$$\delta_H = \frac{\mu B_H}{1+\mu B_H}, \tag{3}$$

where $M=\lambda+2\mu$, and both λ and μ are Lamé parameters. These are fractional parameters which range from 0 to 1. In all cases when the fracture weaknesses are zero the fracture has no influence on the total compliance.

Stiffness matrix for a single vertical fracture normal to the x-axis

The stiffness matrix for a single vertical fracture perpendicular to the x-axis in a background isotropic media is (Schoenberg & Douma, 1998)

$$\mathbf{C} = \begin{bmatrix} M(1-\delta_N) & \lambda(1-\delta_N) & \lambda(1-\delta_N) & 0 & 0 & 0 \\ \lambda(1-\delta_N) & M(1-\chi^2\delta_N) & \lambda(1-\chi\delta_N) & 0 & 0 & 0 \\ \lambda(1-\delta_N) & \lambda(1-\chi\delta_N) & M(1-\chi^2\delta_N) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu(1-\delta_V) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu(1-\delta_H) \end{bmatrix}, \quad (4)$$

where χ=1-2g and g is the square of the S-wave velocity, β, to P-wave velocity, α, ratio of the background isotropic rock. Equation (4) can be written more compactly in symbolic matrix notation

$$\mathbf{C} = \mathbf{S}^{-1} = \mathbf{C}_b - \mathbf{C}_f, \quad (5)$$

where $\mathbf{C}_b$ is the isotropic background stiffness matrix and $\mathbf{C}_f$ is the stiffness matrix describing the perturbation due to the fractures.

Stiffness matrix for a single dipping fracture

In order to introduce fracture dip, the stiffness matrix (equation 5) may be rotated about the y-axis using a Bond transformation (Winterstein, 1990)

$$\tilde{\mathbf{C}} = \mathbf{MCM}^T = \mathbf{M}(\mathbf{C}_b - \mathbf{C}_f)\mathbf{M}^T, \quad (6)$$

where **M** is the bond transform about the y-axis (Carcione, 2001). Since the isotropic background is invariant under rotation

$$\tilde{\mathbf{C}} = \mathbf{C}_b - \mathbf{MC}_f\mathbf{M}^T, \quad (7)$$

only the fracture perturbation stiffness matrix needs to be rotated, the result of which is listed in the Appendix.

Reflectivity expressions

In order to emphasize the azimuthal dependence of P-wave Amplitude versus Azimuth, ϕ, Downton et al. (2011) rewrote the linearized AVAz expression of Pšenčik and Martins (2001) for general weakly anisotropic media as the truncated Fourier series

$$R(\phi,\theta) = r_0(\theta) + r_2(\theta)\cos(2\phi) + r_4(\theta)\cos(4\phi), \quad (8)$$

where

$$r_0(\theta) = w_{00} + w_{01}\sin^2(\theta) + w_{02}\sin^2(\theta)\tan^2(\theta), \quad (9)$$

$$r_2(\theta) = w_{12}\sin^2\theta + w_{22}\sin^2\theta\tan^2\theta, \quad (10)$$

$$r_4(\theta) = w_{24}\sin^2\theta\tan^2\theta, \quad (11)$$

and where the w_{ij} coefficients are defined in Downton et al. (2011) Appendix B. The FCs r_n are each dependent on the average angle of incidence θ. Before going into the details of the w_{ij} coefficient calculations, there are several general observations that can be made. The n=0 FC (equation 9) describes the DC (or average amplitude) and has the same form as the classic 3-term AVO expression where the w_{00}, w_{01}, and w_{02} are modified from the isotropic expression by the presence of the fractures. The n=4 FCs are angle dependent functions of a single variable. The n=2 FCs have the most complex form, being a function of two variables.

Until now, the fracture strike has not been discussed. The problem was originally posed so that the normal of the fracture was the x-axis. In order to generalize this to any orientation, the fracture may be rotated anticlockwise ϕ_{sym} about the z-axis by performing a change of variables $\phi \rightarrow (\phi - \phi_{sym})$ so that equation (8) becomes

$$R(\phi,\theta) = r_0(\theta) + r_2(\theta)\cos(2(\phi - \phi_{sym})) + r_4(\theta)\cos(4(\phi - \phi_{sym})). \quad (12)$$

Having established the general form of the solution, the analytic expressions for each of the w_{ij} coefficients are next derived. This involves substituting the density ρ normalized stiffness matrix elements (Appendix) into Appendix B of Downton et al. (2011). Defining the dip ψ from the vertical and the isotropic gradient

$$B_{iso} = \left(\frac{\Delta\alpha}{2\overline{\alpha}}\right) - 4\frac{\beta^2}{\alpha^2}\left(\frac{\Delta\beta}{\beta} - \frac{\Delta\rho}{2\rho}\right), \tag{11}$$

the first 3 coefficients, which control the AVO behavior, are

$$w_{00} = \left(\frac{\Delta\alpha}{2\overline{\alpha}} + \frac{\Delta\rho}{2\overline{\rho}}\right) - \frac{1}{4}\left(1 - 2g\cos^2\psi\right)^2 \Delta\delta_N - \cos^2\psi\sin^2\psi\left(g\Delta\delta_V\right), \tag{12}$$

$$w_{01} = B_{iso} - \frac{1}{4}\left(\chi - 2g\sin^2\psi\right)\Delta\delta_N + \frac{1}{2}\left(g\Delta\delta_V\right) + \frac{1}{2}\left(\sin^2\psi\right)\left(g\Delta\delta_H\right), \tag{13}$$

$$\begin{aligned} w_{02} = \frac{\Delta\alpha}{2\overline{\alpha}} - \frac{1}{4}\left[\chi^2 + 2g\left(\chi + \frac{3}{4}g\cos^2\psi\right)\cos^2\psi\right]\Delta\delta_N \\ - \frac{3}{8}\sin^2\psi\cos^2\psi\left(g\Delta\delta_V\right) - \frac{1}{8}\cos^2\psi\left(g\Delta\delta_H\right). \end{aligned} \tag{14}$$

Note that these expressions are written in terms of the change in fracture weakness parameters between the two layers (i.e. $\Delta\delta_N$, $\Delta\delta_V$, $\Delta\delta_H$). The remaining 3 coefficients control the amplitude versus azimuth. Rather than write these in terms of fracture weaknesses, it is advantageous to rewrite them in terms of weighted differences of fracture weaknesses. These difference parameters turn out to be better resolved as they have smaller uncertainty. For asymmetric fractures the anisotropic gradient is

$$B_{ani} = g\left(\Delta\delta_V - \chi\Delta\delta_N\right). \tag{15}$$

Bakulin et al. (2001) shows that for a rotationally invariant fracture this results in the same definition of the anisotropic gradient that Rüger uses. In addition to the anisotropic gradient, two other weighted differences

$$\kappa_V = g\left(\Delta\delta_V - g\Delta\delta_N\right), \tag{17}$$

$$\kappa_H = g\left(\Delta\delta_H - g\Delta\delta_N\right), \tag{18}$$

prove useful. Having established these three definitions the remaining coefficients are

$$w_{12} = \frac{1}{2}B_{ani}\cos^2\psi - \frac{1}{2}\left(\kappa_H + \kappa_V\cos 2\psi\right)\sin^2\psi, \tag{19}$$

$$w_{22} = \frac{\cos^2\psi}{2}\left(\frac{1-g}{1-3g}\right)B_{ani} - \frac{\cos^2\psi}{2}\left(\left(\frac{1-g}{1-3g}\right) + \sin^2\psi\right)\kappa_v, \tag{20}$$

and

$$w_{24} = \frac{1}{8}\cos^2\psi\left(\kappa_H - \kappa_V\sin^2\psi\right). \tag{21}$$

Note that the w_{12} and w_{22} coefficients control the n=2 periodicity of the AVAz reflectivity while the w_{24} coefficient controls the n=4 periodicity of the AVAz reflectivity.

Rotationally invariant fractures

In the case of rotationally invariant fractures $\kappa = \kappa_V = \kappa_H$ equations (19), (20) and (21) simplify to

$$w_{12} = \cos^2\psi\left(\frac{1}{2}B_{ani} - \kappa\sin^2\psi\right), \tag{22}$$

$$w_{22} = \frac{\cos^2\psi}{2}\left(\left(\frac{1-g}{1-3g}\right)B_{ani} - \left(\left(\frac{1-g}{1-3g}\right) + \sin^2\psi\right)\kappa\right), \tag{23}$$

and

$$w_{24} = \frac{1}{8}\kappa\cos^4\psi. \tag{24}$$

In this simpler case it is obvious that the dip scales each of the terms by the common scalar $\cos^2\psi$. A dip of 30 degrees (from vertical) scales each of the coefficients down by at least 25% from the zero dip case. In actual fact the coefficients get scaled down more than this which is the topic of the next section.

Dip dependence

Let us first focus on the 2nd FC. At near angles of incidence and for vertical fractures Downton (2010) showed that the 2nd FC gives an estimate of the B_{ani} similar to the near offset Rüger equation. Figure 1a shows how the 2nd FC varies as a functions of dip for an average incidence angle of 30 degrees. This is generated for a model where the $\Delta\delta_N$=0.15, $\Delta\delta_V$ =0.20, $\Delta\delta_H$ =0.15 and the Vp/Vs ratio=1.7. The 2nd FC (shown as the black curve) decreases as a function of dip. The 2nd FC equation (10) is a weighted sum of the w_{12} and w_{22} coefficients (equations 19 and 20). For small angles of incidence the w_{12} coefficient dominates. This is clearly evident in Figure 1a where the $w_{12}\sin^2\theta$ term is shown in blue and the $w_{22}\sin^2\theta\tan^2\theta$ term is shown in red. The 2nd FC decreases more than the scalar $\cos^2\psi$ predicts. This is due to the contribution of the negative 2nd term in w_{12} which becomes more important as the dip becomes larger. Because of this extra contribution the 2nd FC approaches zero for a dip about 50 degrees. Since the 2nd FC coefficient is often interpreted as a fracture indicator the fact it may go to zero at some dip is problematic.

Figure 1a is displayed for an average incidence angle of 30 degrees which is within the near offset/angle approximation. In contrast Figure 1b is shown for an incident angle of 45 degrees. In this case the $\sin^2\theta\cos^2\theta$ terms are also important as evidenced by the relative weighting of the w_{12} and w_{22} coefficients in the total response. In the case of the 45 degree data the sign of the 2nd FC can actually switch.

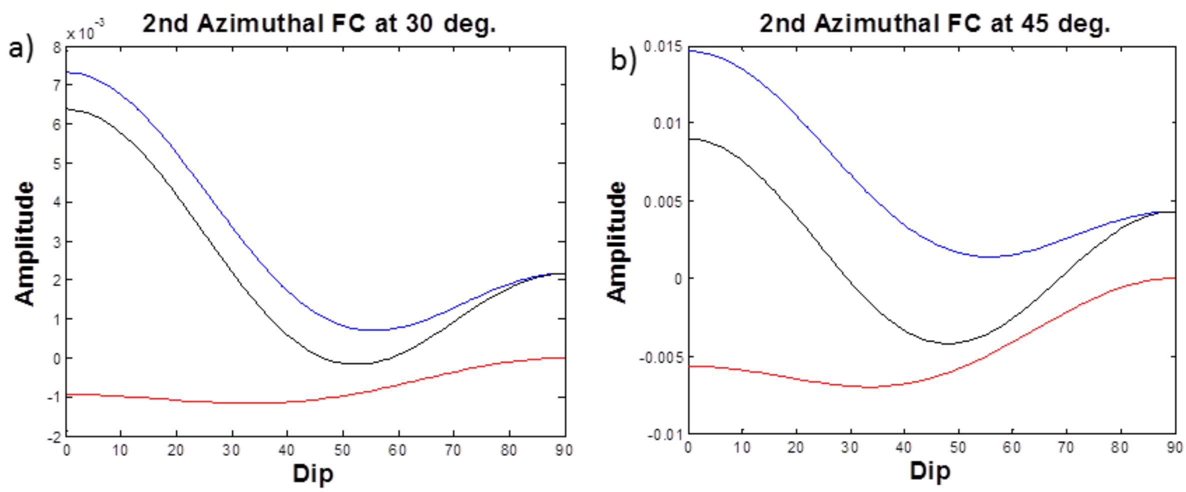

Figure 1: The effect of dip on the 2nd FC at 30 degrees (a) and 45 degrees (b) incidence angle. The blue curve shows the contribution of the w_{12} term while the red curve shows the contribution of the w_{22} term. The total or combined response is shown in black. For small angles (i.e. 30 degrees) the w_{12} term dominates.

Inversion

This last section discusses how to solve the inverse problem and the uniqueness of this solution. For simplicity, this discussion assumes the strike of the fracture is known. This reduces the number of azimuthal bases w_{ij} (or data) from 9 to 6. Downton et al. (2011) discuss how to solve for this in the case of rotationally invariant vertical fractures.

Equations (8), (9), (10), and (11) describe a set of linear equations which form the basis of the inversion. Downton et al. (2011) describe how for different angle data, the azimuthal FCs can be determined. After determining the FCs for at least 3 distinct angle ranges, the coefficients w_{00}, w_{01}, w_{02}, w_{12}, w_{22}, and w_{24} can be determined. This is the first step, since the w_{ij} coefficients are descriptive parameters and give limited insight into the more fundamental rock physical parameters. In order to obtain this understanding we must perform another inversion based on equations (12), (13), (14), (19), (20), and (21) which describe the functional relationships between the w_{ij} coefficients and the fractional elastic and fracture parameters.

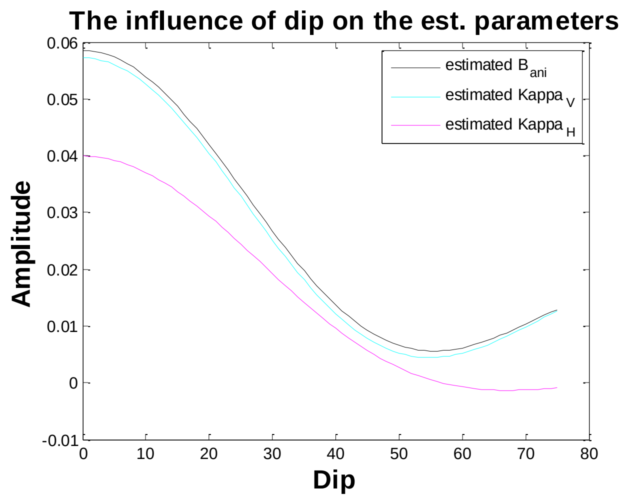

Figure 2: Inversion estimates of B_{ani}, κ_V and κ_H assuming the fractures are vertical. The horizontal axis displays the actual dip of the fractures. When the dip is 0 degrees from the vertical the estimate corresponds to the ideal solution. Non-zero dip introduces bias into the estimates (compared to ideal solution on the y-axis).

These nonlinear equations relate the six w_{ij} coefficients to the three fractional elastic parameters $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, $\Delta\rho/\rho$ and four fractional fracture weakness parameters $\Delta\delta_N$, $\Delta\delta_V$, $\Delta\delta_H$ and dip ψ. Hence the problem is underdetermined since there are seven unknowns to estimate but only six known data. In order to solve this, we must specify some *a priori* information. Two possible simplifications are; 1) assume the fracture is rotationally invariant ($\Delta\delta_V = \Delta\delta_H$) or 2) assume the fractures are vertical $\psi=0$. Historically, it has been assumed that the fractures are vertical. If this assumption is made, then there are six linear equations in six unknowns and the system of equations may be solved exactly. For simplicity, the 3 equations which govern the azimuthal behavior (equations 19, 20, and 21) are only used

$$\begin{bmatrix} w_{12} \\ w_{22} \\ w_{24} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\cos^2\psi & -\frac{1}{2}\left(\cos 2\psi\right)\left(\sin^2\psi\right) & -\frac{1}{2}\sin^2\psi \\ \frac{1}{2}\cos^2\psi\left(\frac{1-g}{1-3g}\right) & -\frac{1}{2}\cos^2\psi\left(\frac{1-g}{1-3g}+\sin^2\psi\right) & 0 \\ 0 & -\frac{1}{8}\left(\cos^2\psi\right)\left(\sin^2\psi\right) & \frac{1}{8}\cos^2\psi \end{bmatrix} \begin{bmatrix} B_{ani} \\ \kappa_V \\ \kappa_H \end{bmatrix}. \quad (24)$$

Given g and specifying $\psi=0$ this set of equations may be solved exactly

$$\begin{bmatrix} B_{ani} \\ \kappa_V \\ \kappa_H \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 2 & -\frac{2}{\gamma} & 0 \\ 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} w_{12} \\ w_{22} \\ w_{24} \end{bmatrix}, \quad (25)$$

where ν=(1-g)/(1-3g). If in fact the fractures are dipping, this will result in biased estimates. In order to quantify this, I created a series of models with dips varying from 0 to 75 degrees using the same model as specified earlier. Equation (25) is then inverted to estimate B_{ani}, κ_V and $\kappa_{H.}$ Figure 2 shows the results. The solution for the zero dip model gives the correct answer (the solution along the vertical axis). The solutions for nonzero dip are all biased. The bias becomes significant for dips greater than 30 degrees.

Conclusions

This paper derived a linearized expression describing how the P-wave amplitude varies as a function of angle of incidence and azimuth for a single dipping asymmetric fracture described in terms of the linear slip fracture weakness parameters. The influence of dip can be studied by examining the relative simple form of this analytic expression. To a first order, the dip dampens the azimuthal response by the square of the cosine of the dip, but in practise the dampening is slightly greater due to 2nd order effects. The linearized expression provides a relatively simple set of equations to perform the inverse problem. Upon examining these equations it is evident that the inverse problem is underdetermined, requiring *a priori* data or assumptions in order to solve the problem in a stable fashion. A modeling study was performed to understand the bias that the assumption of vertical fractures would introduce to the estimates. It was found that dips greater than 30 degrees introduce significant bias.

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Appendix

The non-zero density normalized stiffness matrix elements for the dipping fracture perturbation matrix $\tilde{\mathbf{A}}_f = \mathbf{M}\mathbf{C}_f\mathbf{M}^T / \rho$ are

$$a_f^{(11/33)} = \alpha^2\delta_N\left[\frac{1}{8}\left(3+2\chi+3\chi^2\right)\pm\frac{1}{2}\left(1-\chi^2\right)\cos 2\psi+\frac{1}{8}(1-\chi)^2\cos 4\psi\right]+\delta_V\beta^2\left[\frac{1}{2}-\frac{1}{2}\cos 4\psi\right], \qquad \text{(A-1)}$$

$$a_{f_N}^{(22)} = \alpha^2\delta_N\chi^2, \qquad \text{(A-2)}$$

$$a_f^{(12/23)} = \alpha^2 \delta_N \left[\frac{1}{2} \chi (1+\chi) \pm \frac{1}{2} \chi (1-\chi) \cos 2\psi \right], \tag{A-3}$$

$$a_f^{(13)} = \alpha^2 \delta_N \left[\frac{1}{8} \left(1 + 6\chi + \chi^2\right) - \frac{1}{8} (1-\chi)^2 \cos 4\psi \right] - \delta_V \beta^2 \left[\frac{1}{2} - \frac{1}{2} \cos 4\psi \right], \tag{A-4}$$

$$a_f^{(51/53)} = \alpha^2 \delta_N \left[\frac{1}{4} \left(1 - \chi^2\right) \sin 2\psi \pm \frac{1}{8} (1-\chi)^2 \sin 4\psi \right] \mp \frac{1}{2} \delta_V \beta^2 \sin 4\psi, \tag{A-5}$$

$$a_f^{(52)} = \frac{1}{2} \alpha^2 \delta_N \chi (1-\chi) \sin 2\psi, \tag{A-6}$$

$$a_f^{(55)} = \alpha^2 \delta_N \left[\frac{1}{8} (1-\chi)^2 - \frac{1}{8} (1-\chi)^2 \cos 4\psi \right] + \delta_V \beta^2 \left[\frac{1}{2} + \frac{1}{2} \cos 4\psi \right], \tag{A-7}$$

$$a_f^{(44/66)} = \beta^2 - \delta_H \beta^2 \left[\frac{1}{2} \mp \frac{1}{2} \cos 2\psi \right], \tag{A-8}$$

and

$$a_f^{(64)} = \frac{1}{2} \delta_H \beta^2 \sin 2\psi. \tag{A-9}$$