Regular grids travel time calculation – a practical Huygens wavefront expansion approach

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Summary

We present a practical regular grid wave traveltime calculation that is based on Huygens wavefront expansion and grid traveltime mapping. Wavefront expansion is carried out by finite difference approximations to the equations that are equivalent to the Eikonal equation with a fixed time interval. Mapping traveltime from wavefront to regular grids is based on a dynamic ray tracing paraxial approximation.

Introduction

A regular grid model is often used in real seismic data processing, e.g. tomography and migration, and so the calculation of wave travel time from a point source on a regular grid is of great interest. Though traveltime computation is widely used in seismic modeling and imaging, attaining sufficient accuracy without compromising speed and robustness is often a problem. Moreover, there is no easy way to obtain the traveltimes corresponding to the multiple arrivals that appear in complex velocity media. The trade-off between speed and accuracy becomes apparent in the choice between the two most commonly used methods: ray tracing and numerical solutions to the eikonal equation.

Finite difference eikonal solvers provide a relatively fast and robust method of traveltime computations (e.g. Vidale, 1988; Sethian and Popovici, 1999). With this method, the wavefront can be tracked approximately via a layer setting technique following the causality of wave propagation and efficiencies can be obtained by using heap sorting (e.g. Cao and Greenhalgh 1994). Because this method is directly calculating traveltime on grids, it avoids the problem of traveltime interpolation from wavefront to regular grids. However, this eikonal solver can only compute first-arrival traveltimes and the inability to track wavefronts arriving at later times is a drawback of this method. In complex velocity structures, the first arrivals do not necessarily correspond to the most energetic waves, and other arrivals can be crucially important for accurate modeling and imaging (Geoltrain and Brac, 1993). Moreover, there is still a need to improve the accuracy of the finite difference solution (Yao, Galbraith and Kolesar, 2013).

Grid traveltime calculation based on ray tracing is another common option for regular grids (e.g. Ettrich and Gajeweki, 1996). The basic idea of this method is that, at each time step, rays with different ray parameters are traced and a new wavefront is constructed by connecting the end points from rays. Each wavefront defines an isochronic traveltime curve and the traveltime on a regular grid can be obtained by mapping from the wavefront after wavefronts sweep the whole model space. While this method has the advantage of tracking multi-arrivals and the benefits of accuracy from ray tracing, it is computationally inefficient and not very stable when tracing rays through complex geological structures.

In this paper, we propose a practical regular grid travel time calculation based on Huygens wavefront expansion (e.g. Sava and Fomel, 2001). Huygens wavefront expansion is a numerical solution to the eikonal equation formulated in a ray coordinate system. Because it shares the information between rays, unlike ray tracing where tracing each ray is individually independent, it is robust and stable for a complex geological model. Mapping traveltimes from wavefront to grid can be carried out by a simple formula derived from a paraxial approximation.

Brief description of Huygens wavefront expansion

Assuming that points $x(\tau,\gamma), z(\tau,\gamma)$ are located on a wave propagation wavefront and that they satisfy

$$\frac{\partial x}{\partial \tau}\frac{\partial x}{\partial \gamma}+\frac{\partial z}{\partial \tau}\frac{\partial z}{\partial \gamma}=0 \qquad (1)$$

where x and z are spatial coordinates, τ is the traveltime (eikonal), $\gamma(x,y)$ is a parameter that defines the wave propagation direction. Equation (1) simply tells us that the wave propagation direction is perpendicular locally to the wavefront. With this constraint, we can write wavefront extrapolation as a family of

$$\left(\left(\frac{x-x(\tau,\gamma)}{\Delta t}\right)\right)^2+\left(\left(\frac{z-z(\tau,\gamma)}{\Delta t}\right)\right)^2=v^2(x(\tau,\gamma),z(\tau,\gamma))$$

or

$$(x-x(\tau,\gamma))^2+(z-z(\tau,\gamma))^2=v^2\Delta t=r^2(\tau,\gamma) \qquad (2)$$

Equation (2) is simply the equation of a circle with center $(x(\tau,\gamma),z(\tau,\gamma))$ located on the current wavefront. The new wavefront is extrapolated from a previous wavefront by solving this equation, i.e. an envelope of points x and y, which is the physical meaning of the Huygens principle.

Considering a family of Huygens circles, centered at points along the current wavefront and with the first order discrete finite difference approximation, the propagated wavefront defined by spatial coordinates $x(\tau,\gamma)\ and\ z(\tau,\gamma)$ can be written as (e.g. Sava and Fomel, 2001)

$$\begin{aligned} x_{j+1}^i &= x_j^i - r_j^i(\alpha(x_j^{i+1}-x_j^{i-1}) \pm \beta(z_j^{i+1}-z_j^{i-1}) \\ z_{j+1}^i &= z_j^i - r_j^i(\alpha(z_j^{i+1}-z_j^{i-1}) \mp \beta(x_j^{i+1}-x_j^{i-1}) \end{aligned} \qquad (3)$$

Where

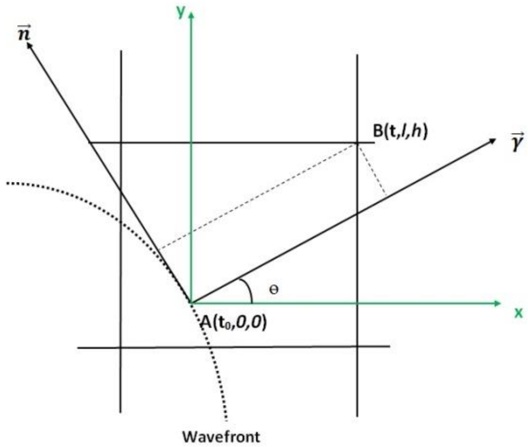
$$\alpha=\frac{r_j^{i+1}-r_j^{i-1}}{\left(x_j^{i+1}-x_j^{i-1}\right)^2+\left(z_j^{i+1}-z_j^{i-1}\right)^2}$$

$$\beta=sign(x_j^{i+1}-x_j^{i-1})\frac{\sqrt{\left(x_j^{i+1}-x_j^{i-1}\right)^2+\left(z_j^{i+1}-z_j^{i-1}\right)^2+\left(r_j^{i+1}-r_j^{i-1}\right)^2}}{\left(x_j^{i+1}-x_j^{i-1}\right)^2+\left(z_j^{i+1}-z_j^{i-1}\right)^2}$$

In the equations above, the index i corresponds to the ray parameter γ, and j corresponds to the traveltime τ. Equation (5) updates the wavefront with an explicit finite difference scheme. For wavefront tracing in inhomogeneous media, this method is much more computationally efficient than traditional ray codes (Sava and Fomel, 2001). It is also robust because the information between rays is shared for each update of $x(\tau,\gamma)\ and\ z(\tau,\gamma)$. Unlike many other eikonal solvers, this method is actually carried out in ray coordinates instead of model Cartesian coordinate. Traveltime that could be multi-valued in a model Cartesian coordinate system is now single valued in ray coordinates and therefore, equation (3) has the ability to track multiple arrivals.

Regular grids travetime mapping

With equation (3), the evolving wavefront can be forward tracked with each time step and therefore, known traveltimes on the wavefronts span 2D space. However, as the wavefront is discretely expanding, it may not necessarily pass through grid points. Therefore, we need to map the traveltime from wavefronts to grid points. The mapping is based on a dynamic ray tracing technique. For example, in Figure 1, if the known traveltime at the point A on the wavefront is time t_0, then at point B on a grid with local ray coordinate *(l,h)*, the traveltime can be approximated as (e.g. Cerveny, 2001)



Fiugre 1. Vicinity travel time extrapolation, where $\vec{\gamma}$ is wave propagation direction and $\vec{n}$ is tangent direction of wavefront. Traveltime t at grid point B is extrapolated from known traveltime t_0 at point A on wavefront.

$$t(l,h) \approx (t_0 + \frac{l}{v}) - \frac{l}{v^2}[(\nabla v \cdot \vec{n})h + \frac{1}{2}(\nabla v \cdot \vec{\gamma})l] \quad (4)$$

where, $\vec{n}$ is a unit vector perpendicular to ray direction vector $\vec{\gamma}$. In the right side of equation (4), the first term is the plane wave approximation and the second term is a curvature correction.
With further coordinate rotation transform,

$$h = (x - x_0)cos\theta - (z - z_0)sin\theta$$
$$l = (x - x_0)sin\theta + (z - z_0)cos\theta \quad (5)$$

and

$$\nabla v \cdot \vec{n} = v_x \cos\theta - v_z \sin\theta$$
$$\nabla v \cdot \vec{\gamma} = v_x \sin\theta + v_z \cos\theta \quad (6)$$

we can obtain the calculation of traveltime on grids in Cartesian x-y coordinates. For instance, if the variation of velocity is smooth the second term can be ignored and then a simple plane wave approximation is

$$t(x,y) \approx t_0 + \frac{1}{v}((x - x_0)sin\theta + (z - z_0)cos\theta) \quad (7)$$

Examples

We use two numerical examples to demonstrate how our algorithm works. First we propose a 100 by 100 2D homogeneous grid model where parameters for both velocity and grid size are one unit. With this model, the calculated error on the grid will be exaggerated. Figure 2 shows the absolute error distribution with a value of 0.02 put on the source location for scaling purposes. From the figure we can see that the maximum absolute error is about 0.01 related to a traveltime around 90. Our second example is a velocity linearly increasing with depth where v_0 is 100 and the velocity gradient is 2. The grid size is also one unit. The result is shown in figure 3, where the ray path is shown by green lines. Contours for the wavefronts start at 0.1, increasing to 0.8 with steps of 0.1. Red curves represent true wavefront locations and dark curves represent calculated contours generated from regular grid traveltimes.

Conclusions

We presented a practical regular grid wave traveltime calculation that is based on Huygens wavefront expansion and vicinity ray travel time approximation. The finite difference Huygens wavefront has the advantages of efficiency, accuracy and computational stability; vicinity ray travel time approximation provides a simple and accurate mapping formula for grids traveltime calculation.

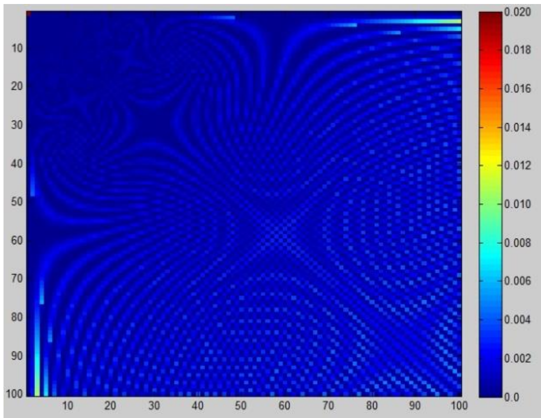


Figure 2. Traveltime error distribution

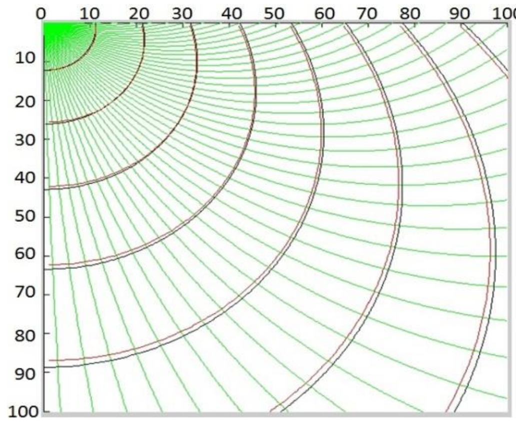


Figure 3. Contours comparison: red curves are theoretic contours and dark curves generated from grid traveltime.

Acknowledgements

Thanks to WesternGeco for permission to present this paper.

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