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A Nuclear Norm Minimization Algorithm with Application to Five Dimensional (5D) Seismic Data Recovery

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Summary

In this paper we present a new algorithm to reconstruct prestack (5D) seismic data. If one considers seismic data at a given frequency and, for instance, in the x midpoint, y midpoint, offset and azimuth domain, the data volume can be represented via a 4th order tensor. Seismic data reconstruction can be posed as a tensor completion problem where it is assumed that the fully sampled data can be represented by a low rank tensor. The alternating direction method of multipliers (ADMM) is utilized to estimate the fully sampled low rank tensor that honours the observations. A field example from a data set from a heavy oil field in Alberta is used to evaluate the proposed tensor completion method.

Introduction

Multidimensional seismic data reconstruction is often tackled via regression methods. The available data are expressed in terms of known basis functions multiplied by coefficients that weight the contribution of each function in the basis to the representation of the signal. Techniques under this category often adopt Fourier basis and seek an expansion in terms of a sparse collection of coefficients. Once these coefficients are found, Fourier synthesis is used to estimate data at unobserved spatial positions (Liu and Sacchi, 2004). Fourier methods driven by sparsity promoting techniques are also connected to the well-established family of methods that operate under the assumption of spatial predictability (Spitz, 1991; Naghizadeh and Sacchi, 2007). This can be easily shown by the equivalence that exists between predictability in space and sparsity in wave-number domains for signals that admit a representation in terms of a superposition of multidimensional plane waves.

In this paper we discuss a multidimensional reconstruction method that is based on the concept of tensor completion. The prestack seismic volume can be represented by four spatial coordinates and frequency. The spatial coordinates in our analysis are x and y midpoints, offset and azimuth (m_x, m_y, h, az). We assume that the fully sampled seismic tensor is a low rank structure. Missing data and noise will increase the rank of the tensor and therefore, rank reduction methods for tensors are adopted to estimate the ideal fully sampled seismic volume. The proposed method resembles reconstruction techniques that operate under the framework of Cadzow or Multichannel Singular Spectrum Analysis (MSSA) reconstruction (Trickett, 2008; Trickett and Burroughs, 2009; Trickett et al., 2010; Oropeza and Sacchi, 2011). However, Cadzow/MSSA reconstruction techniques operate on Hankel matrices that are formed with prestack seismic data. Tensor completion methods, on the other hand, directly operate on the multidimensional volume.

Kreimer and Sacchi (2012) proposed to use the Higher Order Singular Value Decomposition (HOSVD) to recover missing data via the aforementioned concept. In this presentation we examine a new algorithm that permits to write the tensor completion problem in the framework of convex optimization (Gandy et al., 2011). One advantage of the proposed technique is that the rank of the tensor is revealed by the optimization process. The latter is the main difference to our original tensor completion algorithm via HOSVD where the desired rank was provided as an input variable.

Theory

We start our analysis by considering prestack seismic data in the receiver, source and frequency space. For a given monochromatic frequency ω, x and y midpoints, offset and azimuth (m_x, m_y, h, az), the seismic data after binning can be represented by a fourth order tensor. We will denote tensors with bold calligraphic fonts $\boldsymbol{\mathcal{D}}$, matrices with bold capital fonts $\mathbf{D}$ and scalars with italic letters a. The unfoldings of a fourth-order tensor $\boldsymbol{\mathcal{D}}$ will be written as $\mathbf{D}^{(i)}, i = 1,2,3,4$. Four unfoldings exist for a fourth-order tensor and they consist of a re-ordering of its elements into a matrix. The operations of unfolding and folding require careful manipulation of the indices of the tensor and consist of mapping a tensor to a matrix and vice versa. Furthermore, the nuclear norm of a matrix $\mathbf{A}$ is $\|\mathbf{A}\|_* = \sum_{i=1}^{n} \sigma_i$, being σ_i the singular values of the matrix. The nuclear norm of a tensor will be defined as the sum of the nuclear norms of its unfoldings

$$\|\boldsymbol{\mathcal{D}}\|_* = \sum_{i=1}^{4} \|\mathbf{D}^{(i)}\|_* \tag{1}$$

The cost function for the interpolation and denoising problem is

$$\text{minimize} \quad J = \sum_{i=1}^{4} \|\mathbf{D}^{(i)}\|_* + \frac{\lambda}{2} \|\boldsymbol{\mathcal{T}}\boldsymbol{\mathcal{D}} - \boldsymbol{\mathcal{D}}^{obs}\|_F^2, \tag{2}$$

where $\boldsymbol{\mathcal{D}}^{obs}$ are the observations, $\boldsymbol{\mathcal{T}}$ is the sampling operator with the same size as $\boldsymbol{\mathcal{D}}$ and λ is a trade-off parameter. The cost function J is minimized using the alternating direction method of multipliers (ADMM) (Bertsekas and Tsitsiklis, 1989).

Examples

Our field data example is from an orthogonal survey acquired over a heavy oil field in Alberta, Canada. We reconstructed a crossline swath from this survey by dividing into 21 overlapping blocks of inline/crossline (12 inlines overlap length). For each block, the dimensions of the grid were of size $26 \times 26 \times 5 \times 8$, in the inline-crossline-offset-azimuth domain. The azimuth was measured counterclockwise from the East. Furthermore, we used overlapping time windows of 200 samples for the interpolation of each block, where the total number of samples were 1000 (5 windows in total). The bin size in the inline and crossline directions was 5m$\times 5$m, 100m for offset and $45°$ for azimuth. The minimum offset was 50m while the maximum was 450m. The total amount of grid points per block were $26 \times 26 \times 5 \times 8 = 27040$ whereas the total amount of traces in the area were such that 40-45% of the grids were populated. The data were NMO-corrected and low-pass filtered with a cut off frequency of 100Hz prior to the interpolation. The frequencies considered in the algorithm range from 0.1 to 100 Hz, with a sampling rate of 1msec. The running time on a single processor Intel Xeon(R) running at 3.16 Ghz using MATLAB was of approximately 1h for each block. Figure 1 displays the shots and receivers that contribute to the black rectangular area, which is the midpoint area used for the interpolation. We portray a common offset/azimuth gather in Figure 2 for one offset range, a constant crossline and for $az = 22.5°$ and $112.5°$. Furthermore, Figure 3 contains a stack section for a constant crossline before and after reconstruction. Many of the reflections, specially below 0.4s, are denoised after the reconstruction and look continuous, in contrast to what was observed before.

Conclusion

In this article we present prestack seismic data reconstruction and denoising as an inverse problem. The proposed method minimizes the rank of the unfoldings of the tensor in the $F-X$ domain. The rank-reduction operation is carried out directly on the tensor and not in the Hankel matrices generated from the data. This formulation does not require any knowledge about the rank needed for the rank reduction. While we did not display synthetic tests in this article, our experiments showed that this reconstruction scheme can handle events with curvature, as HOSVD (Kreimer and Sacchi, 2012).

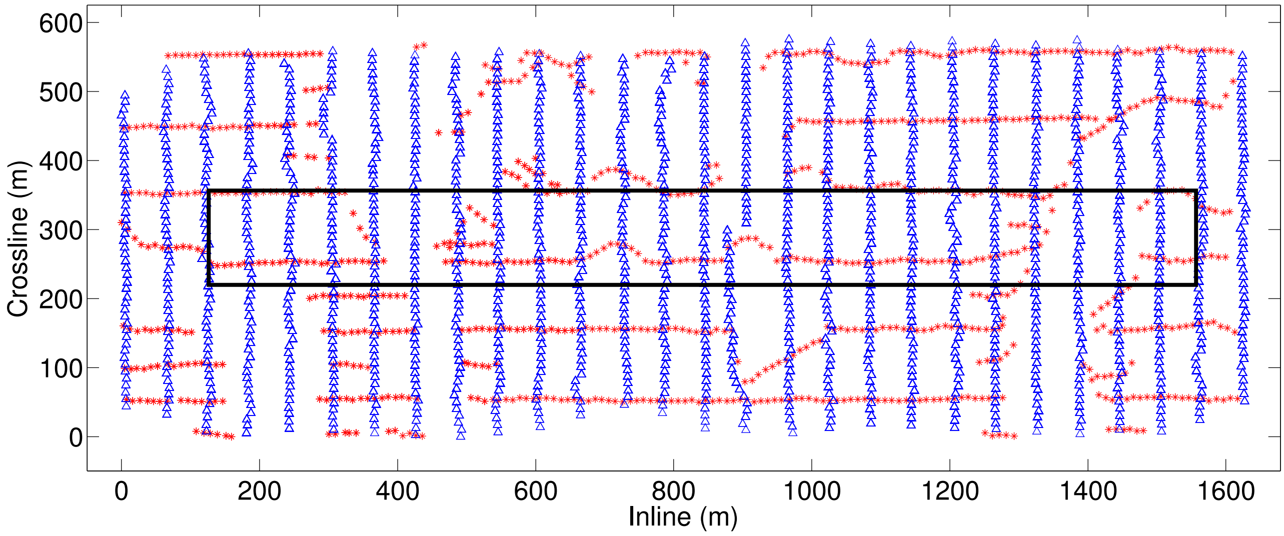

Figure 1: Survey geometry for the real data example, in true aspect ratio. The red stars are the sources and the blue triangles are the receivers. The black square shows the midpoint area used in the reconstruction (comprises all midpoints included in the 21 blocks).

HOVSD and this method are able to reconstruct curved events, unlike Cadzow-based reconstruction or Fourier-based methods. Our land data example demonstrates the performance of the algorithm in a real case scenario. ADMM-based reconstruction presents a formal formulation to the rank-reduction based interpolation problem.

This method does not intend to replace widely used methods in the industry such as MWNI (Liu and Sacchi, 2004). It merely presents a new research path we have started to follow, where we believe that seismic data can be handled in its multidimensional form without adopting matrices for their rank reduction operation. We envision that this sort of approach could lead to other applications in exploration geophysics in the future.

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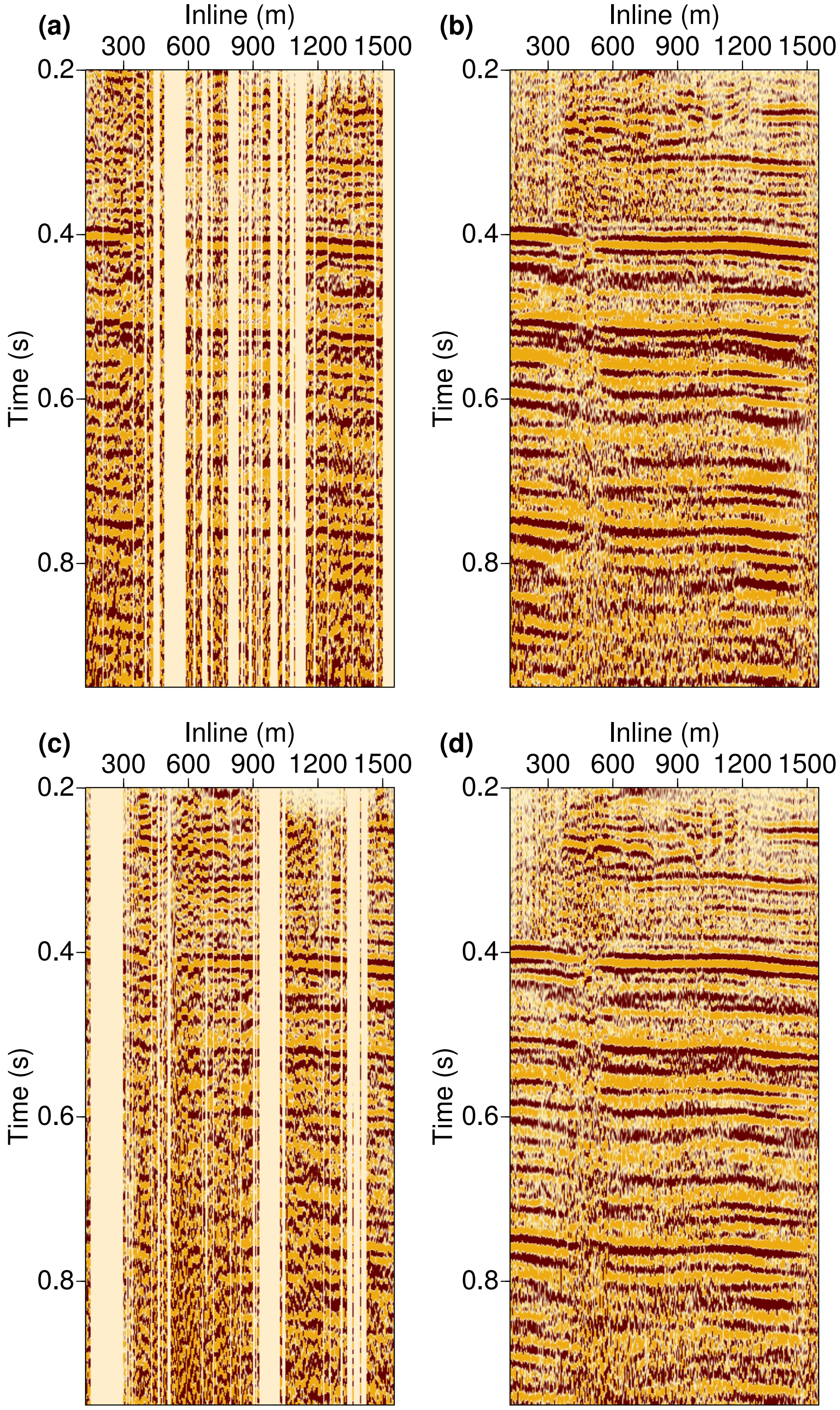

Figure 2: Constant offset (250m) and azimuth displayed for a single crossline. (a) 22.5° azimuth before interpolation. (b) 22.5° azimuth after interpolation. (c) 112.5° azimuth before interpolation. (d) 112.5° after azimuth interpolation.

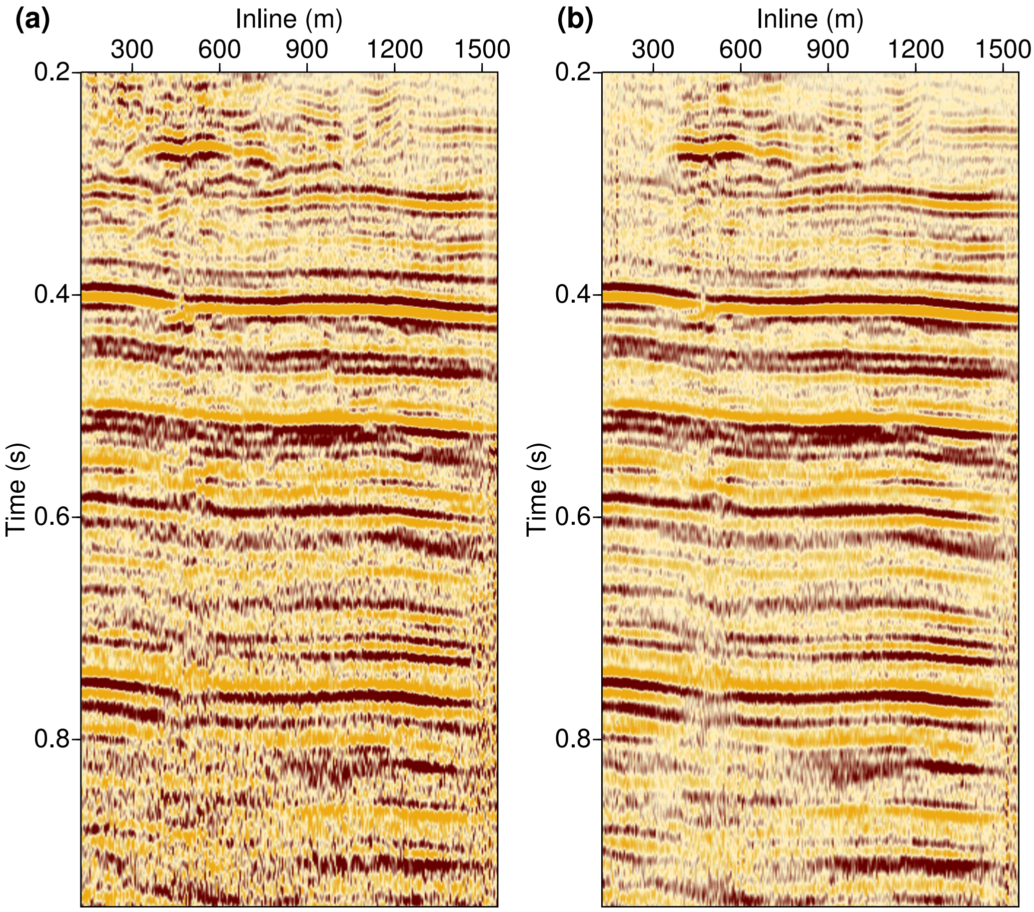

Figure 3: Stack of all 21 blocks for a constant crossline. (a) Before interpolation. (b) After interpolation.