Analysis and Implications of the Archie-Haro Equation in Modeling Resistivity of the Rocks

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Summary

The Archie-Haro equation (modified Archie's equation) theory and accompanying relationships have been proven in the literature (Haro, 2010) by use of the corner angle solution of the Laplace differential equation, which is applicable in electrostatic problems. This approach defines the streamlines that describe the current distribution within the porous system by honoring the tortuosity, topology and conductive-water spread within the rock. Indeed, the openings within the rock frame and conductive paths oblige the electric current to continuously meander and change direction. These changes are quantified by the magnitude of corner angles. The effectiveness of this procedure to calculate water saturation (S_w) has been verified using Archie's and Hamada's core datasets (Haro, 2010).

The Archie-Haro equation solves a problem that consists of obtaining resistivity equivalents at various scales, whilst a substitution of the rock-fluids content occurs (from 100% water to partial water saturation). The solution is built by converting a pore-scale problem to an equivalent cube model (Serra, 1989), and then to well-log scale. The macroscopic Ohm law (referred as simply Ohm's law throughout) can give the equivalents only when the conductive paths are straight. For this reason, calculations require straightening of the curved conductive paths with the Laplace equation first.

Advantages of the proposed theory include:

- It opens the possibility of continuously measuring the tortuosity (m) and saturation (n) exponents with a dipole sonic log (Haro, 2010; Haro, 2006; Mavko, 1998; Mitchell, 1981; Kamel, 1992) and the dielectric-resistivity combination, respectively.

- The new relations establish a mathematical link with the geometry of the rocks.

- It enables us to obtain the solution of more complicated problems, like shaly sands and thin laminated sequences (Haro, 2008).

- The m exponent can be univocally defined at every depth using core data, and n can be expressed as a function of water saturation. Furthermore, there is no need to fix the tortuosity factor (a) equal to 1.

- The number of unknowns reduces from 3 (a, m, n) to only 2: tortuosity (δ) and divergence (ζ) angles.

The Archie equation has been the backbone of petrophysics for more than 70 years. With the proposed enhancements, the Archie equation is no longer empirical but has a sound basis in scientific theory. The Archie-Haro equation enables improved calculations of water saturations, with the concomitant gain in accuracy to estimate hydrocarbons in place.

Introduction

Resistivity modeling of porous media is very controversial. Since its inception in the 1940s, Archie's equation has created lots of concerns and confusion. Although it has been used effectively for years, several authors have identified and reported its shortcomings (Worthington, 1981; Lucia, 1999; Bussian, 1982; Herrick, 1993). It is applicable to clean formations (clay-free) with no conductive solids.

The origin of the equation is purely empirical, with no recognized theoretical basis and no proved link with the geometry of the rocks. Besides, the coefficient and exponents of the equation have shown a broad variation, to the detriment of the calculations.

Many professionals have attempted to improve this state of affairs by adding some rationale to the problem by different means. Theories like electrical efficiency, the equivalent rock element, the graphical model, the connectivity equation, and so forth have emerged as potential solutions.

Previous Archie's-equation theoretical demonstrations have ended with fixed exponents (equal to 1 or 2, depending on the tortuosity relation used) for the porosity (φ) and water saturation terms. Assumptions have usually included the use of spherical, elliptical, or uniformly shaped grains. Modeling has been performed by use of simulators or analytically by applying resistor-equivalent theory. Connectivity has been incorporated in simulators by applying Kirchhoff's laws, which lack information about the direction of the electric current. Ohm's law has usually been utilized in a manner that ignores that a porous medium does not constitute a simple series or parallel system, but a complex resistor network. The exponents *m* and *n* are generally adopted as a fixed value for an entire formation, and the coefficient *a* is assumed equal to 1. Despite the tremendous effort, the ambiguities and incongruities have persisted almost to this day. Haro's 2008 and 2010 papers have shown how to overcome these limitations for shaly and clean rocks, respectively.

Theory and Method

The Laplace differential equation in 2D describes adequately the streamlines and potentials in electrostatic fields without sources (no electrical free charges) in any given orientation. Its corner angle solution contains an exponent that can be associated with the cross section of the current paths (areal porosity, areal saturation). Moreover, the exponent has a mathematical connection with the corner angle, which delineates the electric routes within the porous rock. For these reasons, demonstration of the Archie-Haro equation finally has variable porosity and saturation exponents, as well as direct geometric ties with the conductive trails, as expected. In addition, *a* and *m* are mathematically linked through the corner angle δ (Figure 1), which means that the number of unknowns has been reduced. Now, saturation also has a coefficient *b*, which corresponds to the elongation or tortuosity of the electric paths under partial saturation conditions; *b* is also connected to *n* through ε, another corner angle (Figure 2). The only corresponding unknown is ζ, which is the angle formed by the pore wall and the line joining opposite pore throats. Both *a* and *b* are generally greater than 1. They become equal to 1 when porosity and water saturation are 100% (Haro, 2010), which complies with the corresponding boundary limits. Succinctly, the Archie-Haro equation (1) and additional relationships (2-6) are:

$$S_w{}^n = \frac{abr_w}{\phi^m R_t} \dotfill (1)$$

$$a = \frac{1}{\sin(\delta/2)} \dotfill (2)$$

$$b = \frac{1}{\sin(\varepsilon/2)} \dotfill (3)$$

$$m = \frac{180}{\delta} \dotfill (4)$$

$$n = \frac{180}{\varepsilon} \dotfill (5)$$

$$\tan(\zeta) = \frac{1}{(1 - S_w)\tan(\varepsilon/2)} \dotfill (6)$$

All of the above relationships are oriented. They constitute a system of simultaneous equations to be solved together. Their values change when the direction of the measurement or calculation is changed. This characteristic is a direct result of the anisotropy of the rock. Thus, caution should be exercised when comparing cores and well logs and when using those values for calculations.

Understanding the theory. Water saturation calculation uses only one measurement of porosity and deep resistivity (Rt), despite the fact that thousands of pores exist in the rock at every log depth. The same is true for every parameter of the Archie-Haro equation like a, m, b and n. Therefore, some sort of averaging is needed to transform the data from pore to unit cube scale. Porosity and water saturation are simple linear averages. However, angle averages should honor both Kirchhoff's laws and the significance of the current-streamlines dictated by the Laplace equation. In fact, the streamlines and their corresponding corner angles are quantitatively more important where there is more circulation of the electric current. The following averaging relation applies:

$$\delta = \frac{\sum_0^q \left\{ \left[\sum_0^p |J| \delta_m \right] / p \right\}}{q} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(7)$$

where $|J|$ is the normalized current density magnitude of every streamline, δ_m is the corner angle for every streamline, and p is the number of conductive paths emanating from a junction, all at pore scale; q is the innumerable number of pores at every depth. A pore is the connectivity junction among pores, while a pore throat is the junction within a single pore. The internal summation of Eq. 7 refers to pore scale and accounts for the orientation and relative importance of the conductive paths. It honors Kirchhoff's laws, adequate for pore networks, which signify that the total current flow at a junction should be equal to the summation of the individual contributions. The external summation converts the angles from pore scale to "unit cube" intermediate scale (Haro, 2010).
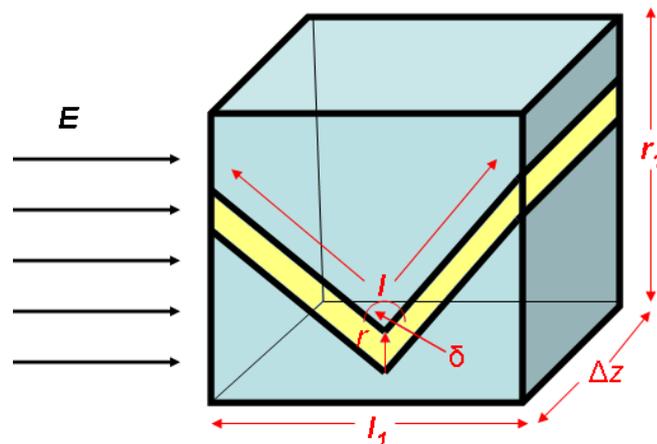

Figure 1: Upscaled model: Unit cube with solid (isolator in light blue) + water with resistivity r_w (in yellow) = Unit cube full of mixture material with equivalent resistivity r_o.

This type of resistivity modeling uses angles; thus, no predetermined shape of the rock grains has to be assumed. Results are applicable to any shape of grains. Due to the averaging, only one porosity value and only one angle value are obtained in the unit cube scale (Fig. 1). This result does not mean that porosity, topology and tortuosity are uniform at pore scale; actual areal porosity and angles vary significantly among the pores and pore throats.

It is well known in core analysis technology that, in order to measure resistance, the core end-faces must be connected with surface electrodes, not with point contact electrodes. The resistance values obtained with these two methods are quite dissimilar because the internal current distribution throughout the core is different. This happens even though the resistivity property of the core and volume of the core obviously do not change. This result is also explained in the basic books of Physics (Halliday and Resnick, 1997) using resistors.

With surface electrodes, the electric current travels straight across resistor rods, and Ohm's law is directly applicable. With a point contact electrode, the problem has to be solved with the Laplace differential equation to properly describe the curved streamlines. This is exactly the effect occurring in every pore (where the pore throats mimic the point electrodes) that forces the current to distort and accommodate to the rock skeleton. It is like having a diffuser and nozzle system (Haro, 2006). If a point electrode is applied transverse to the resistor, the current bends further, and a corner angle of about 90° is created. The resistance changes even more. A similar effect is caused when various pores are interconnected. The streamlines become extremely complicated, if various branches derive from every pore due to topology. In summary, Ohm's Law or resistor laws to obtain equivalents cannot be applied directly in porous media until after employing the most elaborate Laplace solution to straighten the conductive paths. This way, the unit cube model is transformed into the log scale model, which is suitable for log evaluation.
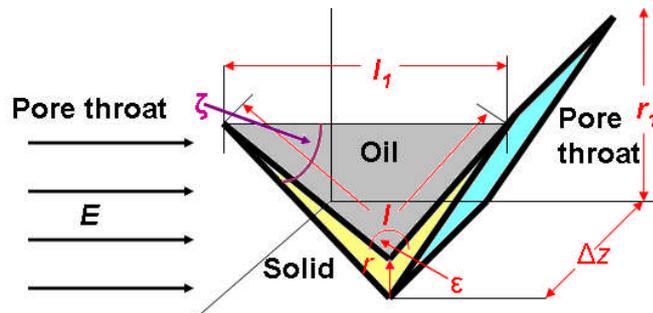

Figure 2: Half pore body upscaled model: Half pore with hydrocarbons (isolator in gray) and water with resistivity r_w (in yellow) = Half pore full of combined material with equivalent resistivity Ør_t. Water distribution reflects water-wet conditions. Notice the elongation of the electric path.

Examples

Core data provide a list of porosity and formation factor (F) values. Since there is not enough data, a is assumed equal to 1 and m is generally back-calculated. This procedure gives a correlation (R^2) of 0.89 between F measured and F calculated for the original Archie dataset. Using equations 2 and 4, a and m can be determined at every point with a correlation coefficient equal to unity. This result confirms the advantage of the new method. The same comment is applicable for finding b and n with equations 3 and 5. In summary, the Archie-Haro model is theoretically sound and fully defined at every depth.

Implementation of the new equation is not difficult. The inputs are angles (δ and ζ), which then can be used to obtain a, b, m, and n, using Eqs. 2 through 6. The last equation expresses n as a function of water saturation. This feature signifies that ε (thus b and n), needs to be obtained by iteration. As reference, δ = 180° applies for fractures. Archie values (a = 1, m = 2) are equivalent to a δ value of 102° +/-17 (Haro, 2010), while δ should decrease as cementation and packing increases and sorting deteriorates. In fact, these characteristics cause an increase in topology/tortuosity. Even lower δ values correspond to non-connected vuggy porosity, because the electric paths enlarge. Round grains should exhibit a value of ζ in the range 45° to 50°. The more elongated the grains, the lower the ζ value, whilst ζ should increase (more than 50°) for oil wet rocks, again due to the elongation of the electric paths.

A Petrophysical analysis example is shown in Fig. 3. A comparison between curve S_{warch} (Archie's S_w with a = 1, m = n = 2) and $S_{warcharo}$ (Archie-Haro's equation) is presented. This well was cored with oil based mud, so reliable S_w core data were available. Water extractions confirmed an r_w (formation water resistivity) value of 1 ohm-m at down hole conditions. The formation is fairly clean, with some shaly sections. An artificially low r_w (0.5 ohm-m) had to be used with Archie's equation to match the core in the clean intervals, whilst the proper r_w was used with the new equation (δ = 88°, ζ = 35°). In addition, Archie's equation is optimistic in the shaly sections because it calculates lower values than core data. Indeed, this has been reported as an observed response of the Archie equation with respect to its own core dataset, which is optimistic for the poor rock, and pessimistic for the good rock (Haro 2010).

The Archie-Haro method exhibits a better and more consistent match. Some differences are observed due to the presence of laminations with both methods.
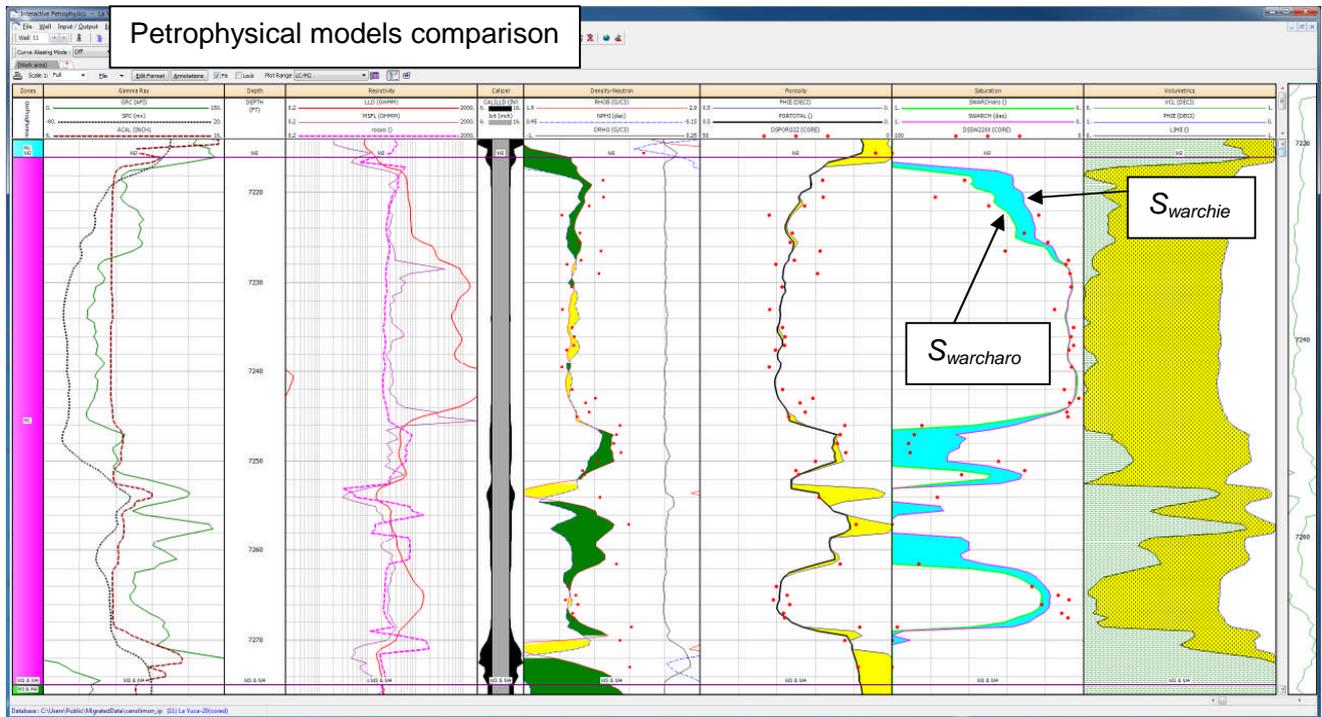

Figure 3: Comparison between the Archie equation results and the Archie-Haro relationships with respect to core data (dots).

Conclusions

The Laplace differential equation and its corner angle solution explain Archie's equation, which has been used successfully for more than 70 years in a variety of petrophysical problems, despite its limitations and shortcomings. The procedure outlined in this paper is the only one that proves variable exponents (*m* and *n*), not fixed or equal to 1 (or two), associated with porosity or water saturation, respectively.

The number of unknowns has diminished from 3 to only 2. The values of *a* and *m*, and *b* and *n* are mathematically linked, and are finally associated with the internal geometry of the rock.

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