Interpolation by Angular Deconvolved prior MWNI (AdMWNI)

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Summary

We propose a new way of applying an angular weight function on the a priori of the Minimum Weighted Norm Interpolation (MWNI) by deconvolving the interpolation model and replacing it by its angular weight function. It is called the angular deconvolved prior MWNI (AdMWNI). This will further stablize the former angular weighted prior MWNI (AwMWNI) method.

Introduction

MWNI has been popularly used as the interpolation engine of 2D and 3D seismic data interpolation in the past few years (Liu and Sacchi, 2004; Trad, 2009). During last GeoConvention, two papers stated that the a priori of the MWNI can significantly impact its results: Chiu (2013) proposed to use an a priori model for the MWNI, which model is derived from performing a local dip scan for a few major dips in the $\boldsymbol{t}$-$\boldsymbol{x}$ time-space domain of the input data. Contrary to that, Ng and Negut (2013) showed that the dip scan can be easily done for all dips in the $\boldsymbol{\omega}$- $\boldsymbol{k_x}$ frequency-wavenumber domain. It is achieved by calculating an angular sum on the input data spectrum, and then by constructing an angular weight function as the dip guide. By multiplying the angular weight function to the input data spectrum, it becomes the a priori model of the MWNI algorithm, i.e. AwMWNI. We found that compared to results obtained by the conventional MWNI algorithm it greatly improved the ability to recover data under more adverse situations such as a block gap or missing data, and upsampling, even for aliased conditions. However, priors obtained from multiplying the angular weight function to the data spectrum may not be good enough estimates for some extremely poor signal coverage situations. Here, we propose the idea of replacing the a priori model amplitude spectrum by the angular weight function.

Theory and Method

Define $d(x,t)$ to be the observed input with missing data, and $D(k_x,\omega)$ its corresponding Fourier transform. Find the angular sum $A(\theta)$ by summing the amplitude values along the radial direction starting from the origin of $D(k_x=0,\omega=0)$ of all usable apparent dip angles θ ; r is the radial length from the origin.

$$A(\theta) = \int_r |D(\theta, r)| dr \qquad (1),$$

$$r = \sqrt{k_x^2 + \omega^2}\ , \theta = \tan^{-1}\frac{k_x}{\omega} \qquad (2).$$

By populating A in the ω- k_x domain, one can construct an angular weight function γ even when the spectral component is aliased wrapping around and beyond the spatial Nyquist wavenumber. This wrapping around feature serves as a dip guide and will protect the aliased dipping reflectors. Keep the amplitude constant in the radial direction but allow it to vary at different angles θ. The angular weight function becomes

$$\gamma(k_x,\omega) = A^p(\theta) \qquad (3).$$

The power p controls the level of emphasis in the presence of linear events, and a zero value places no emphasis (Ng and Negut, 2013). The angular weight function γ becomes an extra dimension not only connecting, but also guiding the dip estimates across all temporal frequency, whereas in the conventional MWNI algorithm, it is totally absent.

In this paper, we propose a new angular weight function γ that contains the inverse spectrum of the input:

$$\gamma(k_x,\omega) = \frac{A^p(\theta)}{\left(\overline{D}^2 + \mu\right)} \qquad (4),$$

where $\overline{D}$ is the slightly smoothed amplitude spatial spectrum of the input observed data d_x , and μ is a prewhitening scalar for tuning control. When μ is a large value, equation (4) degenerates back into equation (3) giving the AwMWNI result (Ng and Negut, 2013). But when μ is a small value as proposed in this paper, equation (4) becomes a deconvolution operator de-emphasizing the 'damaged' input amplitude spectrum due to missing data and emphasizing the angular weight function based on dip construction giving the proposed AdMWNI result. Like AwMWNI, the angular weight function γ is multiplied to the very first a priori model spectrum at every frequency ω for the data fitting in x. This enables the dip impression to guide the solution. As in conventional MWNI, the data fitting is achieved by the conjugate gradient (CG) method imposing sparsity in the transform model m_{kx} .
The adjoint operator giving the transform model at ω is

$$m_{k_x} = \left[TF'\overline{D}\right]^H d_x = \overline{D}FTd_x \qquad (5).$$

The first a priori used is

$$m_{k_x} = \gamma D \qquad (6).$$

Note that if γ =1 or a dip filter, the algorithm degenerates into conventional MWNI.
The forward operator for the approximated data is

$$\tilde{d}_x = \left[TF'\overline{D}\right] m_{k_x} \qquad (7),$$

where m_{kx} is the spatial transform model in k_x , T the sampling diagonal only matrix, F' the inverse spatial Fourier transform operator, F the forward spatial Fourier transform operator, and H the Hermitian transpose.

Both AwMWNI and AdMWNI have one extra interpolation dimension more than conventional MWNI. It is the angular dimension which chains information across all frequencies and thus improves the a priori. Interpolation utilizes a 4D space in x, offset h, ω and θ for 2D prestack data, and a 6D space in x, y, h_x, h_y, ω and θ for 3D prestack data.

Examples:

We tested the ability of various MWNI options in the recovery of a 2D data set that has been highly decimated. The data set is taken from the well-known Benjamin Creek in the Western Canadian foothills, and has been used for the public bench mark purposes in the past.

Figure 1 shows the ‘hidden’ reference full stack of all shots with a shot interval of 100 m, 300 traces per shot giving a stacking fold of 10. CDP interval is 10 m.

Figure 2 shows the stack of 3:1 decimated shots. The shot interval now becomes 300 m which is considered too large for structural data processing and interpretation. A 760 m large data gap appears at the shallow outcrop structural section due to missing shots. Strong linearly dipping shot noise conflicting with complex structural dips is evident everywhere.

Figure 3 shows the stack of the recovery by conventional MWNI, i.e. applying $\gamma = 1$ for equation (6). After interpolation, the fold increased to 30. We used small data blocks (60 CDP traces rolling along, 3 offset groups and 1 s window) for transformation trying to honor structures. Inadequate data coverage caused MWNI to be unstable giving poor and inconsistent results.

Figure 4 shows the stack of the recovery by Angular Weighted prior MWNI (AwMWNI), i.e. using γ in equation (3) for equation (6). The same small data block size was used as before for transformation. The stack shows good and consistent recovery of structures overall; this result is much better than that of the conventional MWNI shown in figure 3. However, at the highlighted shallow large gap, the amplitudes seem too strong.

Figure 5 shows the stack of the recovery by proposed Angular Deconvolved prior MWNI (AdMWNI), i.e. using γ in equation (4) for equation (6). The stack shows good and consistent recovery similar to that of the AwMWNI shown in figure 4. Furthermore, at the circled area where the shallow outcrop structural gap situated, the angular deconvolved prior AdMWNI result shows further improvement over that of the angular weighted prior AwMWNI.

Conclusions

The proposed angular deconvolved prior for MWNI replaces the a priori amplitude spectrum by the data angular weight function. Subsequently in some extremely poor data coverage area, it provides further stabilization in the inversion as compared to the former angular weighted prior MWNI. When data coverage improves, AdMWNI gives similar results to that of AwMWNI. However, both methods give much improved result than that of the conventional MWNI method.

Acknowledgements

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References

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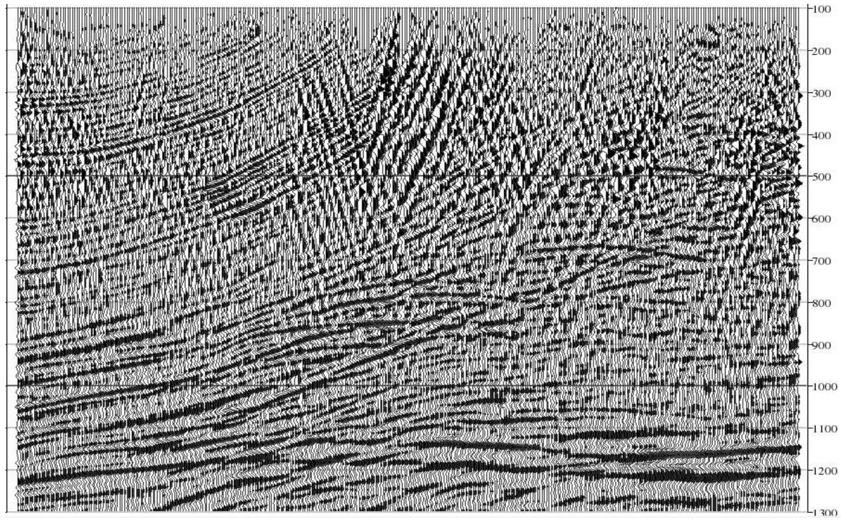

Figure 1. Full stack of all shots - the ‘hidden’ reference. Shot interval 100 m, 30 fold.

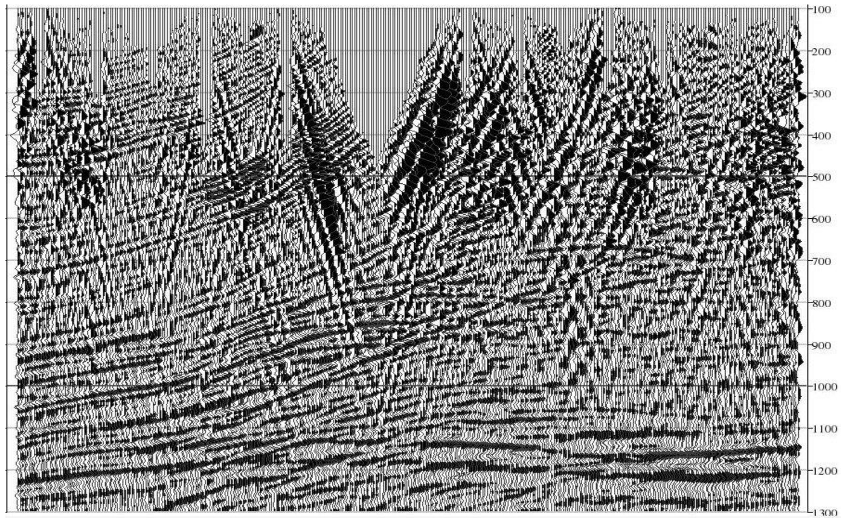

Figure 2. Stack of input 3:1 decimated shots – Very large shot interval 300 m, 10 fold.

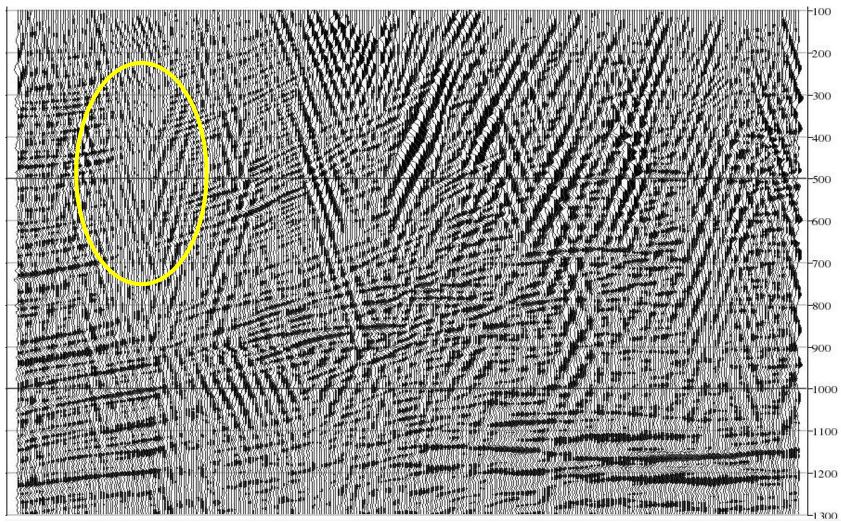

Figure 3. Stack of the recovery by conventional MWNI. Poor quality recovery overall.

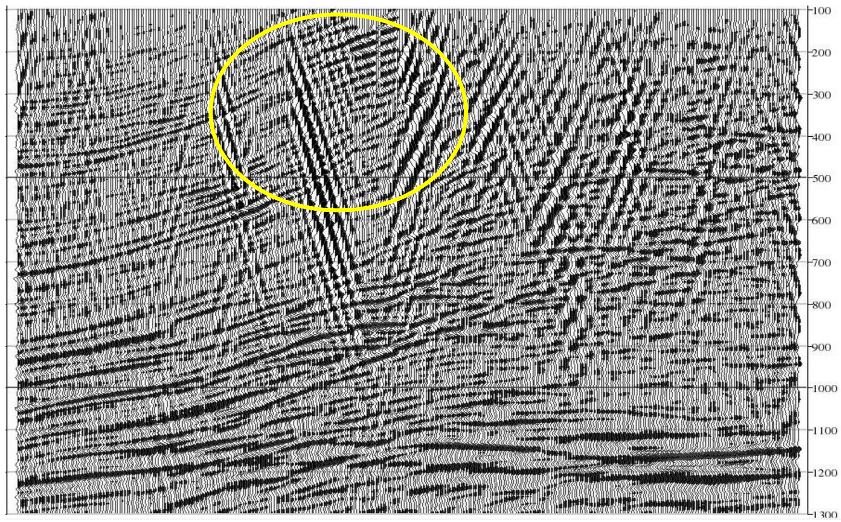

Figure 4. Stack of the recovery by Angular Weighted prior MWNI (AwMWNI).

Good quality recovery overall. At the highlighted big gap, the recovery gives some unrealistic reflectors.

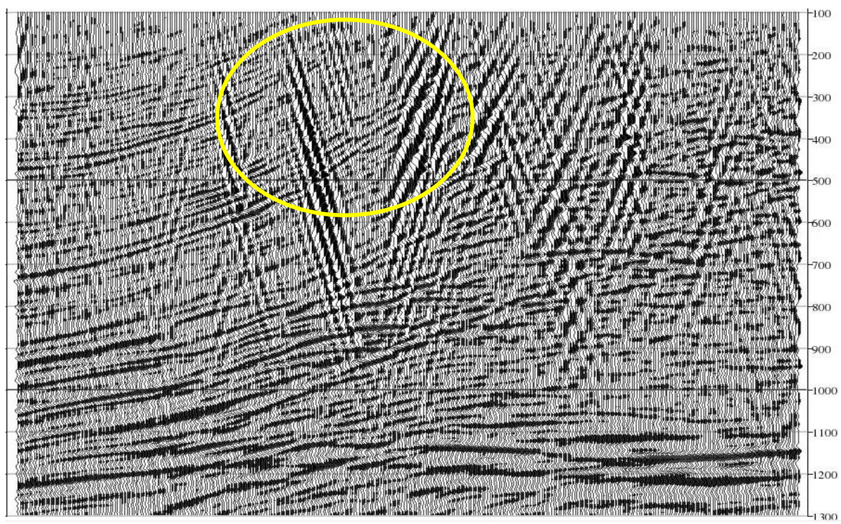

Figure 5. Stack of the recovery by Angular Deconvolved prior MWNI (AdMWNI).

Good quality recovery resolving complex conflicting dips overall. At the highlighted big gap, the recovery shows reflectors that are more reasonable than those by AwMWNI in figure 4.