

## Removing blended sources interference using Stolt operator

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### Summary

We use Stolt migration/de-migration operator to remove blended sources interference in common receiver gathers. Stolt operator is used as a computationally efficient alternative to the apex shifted hyperbolic Radon operator. The problem of estimating the interference free data using Stolt operator is posed as an inversion problem. This inversion utilizes an  $\ell_1$  misfit function that is not susceptible to erratic noise in the data such as blending interferences.

### Introduction

Blended sources acquisition offers many advantages over conventional acquisition such as reducing acquisition time and increasing illumination (Garotta, 1983; Beasley et al., 1998; Berkhout, 2008; Ikelle, 2010). Blended sources data is equivalent to time shifting individual sources data and summing them according to the sources firing times. Therefore, the blended sources data can be represented as a function of the single sources data by the following equation

$$\mathbf{b} = \Gamma \mathbf{D} \quad (1)$$

where  $\mathbf{b}$  is the blended data,  $\mathbf{D}$  represent the original data cube that would be recorded without source overlapping and  $\Gamma$  is the blending operator (Berkhout, 2008). Blended data can be separated by the adjoint of the blending operator (known as *pseudodeblending* operator)

$$\tilde{\mathbf{D}} = \Gamma^H \mathbf{b}, \quad (2)$$

where  $\tilde{\mathbf{D}}$  represents the Pseudodeblended data cube. Pseudodeblending removes sources time delays and divide the long blended data onto its equivalent non-overlapping sources data cube (Figure 1). However, pseudodeblending does not remove interferences resulting from overlapping sources. Therefore, pseudodeblended data cube contains considerable amount of interferences that need to be removed using denoising techniques (Berkhout, 2008; Kim et al., 2009; Huo et al., 2012; Ibrahim and Sacchi, 2013, 2014). Figure 2 shows that the blended sources interferences have different structures in different gathers. While interferences have coherent structure in common shot gather, its structure in common receiver gathers is incoherent. The reason for this incoherency in common receiver gathers is the random delays in sources firing times. Therefore, interferences can be removed by denoising the data in common receiver gather (Berkhout, 2008).

### Stolt operator

Recently, Ibrahim and Sacchi (2013, 2014) proposed using robust inversion of apex shifted hyperbolic Radon (ASHRT) transform to separate blended sources data by removing interferences in common receiver gathers. One major disadvantage of Radon transforms is their high computational cost when performed in time domain. However, Radon transform computational speed can be increased when it is computed in the frequency domain. Unfortunately, the best Radon basis that fit the data use hyperbolic or apex shifted hyperbolic travel times which are time variant and so can not be computed in frequency domain. To address this limitation, Trad (2003) proposed using Stolt mapping operator to compute the ASHRT model. Stolt operator (Stolt, 1978) map the frequency  $\omega$  to wavenumber  $k_z$  in Fourier domain for constant velocity. The adjoint Stolt operator can be written in operator format as a concatenation of three operators as

$$\mathbf{L}^T = \mathbf{FFT}_{k_z, k_x}^{-1} \mathbf{M}_{\omega, k_x}^T \mathbf{FFT}_{t, x}, \quad (3)$$

and the forward operator as

$$\mathbf{L} = \mathbf{FFT}_{\omega, k_x}^{-1} \mathbf{M}_{k_z, k_x} \mathbf{FFT}_{z, x}, \quad (4)$$

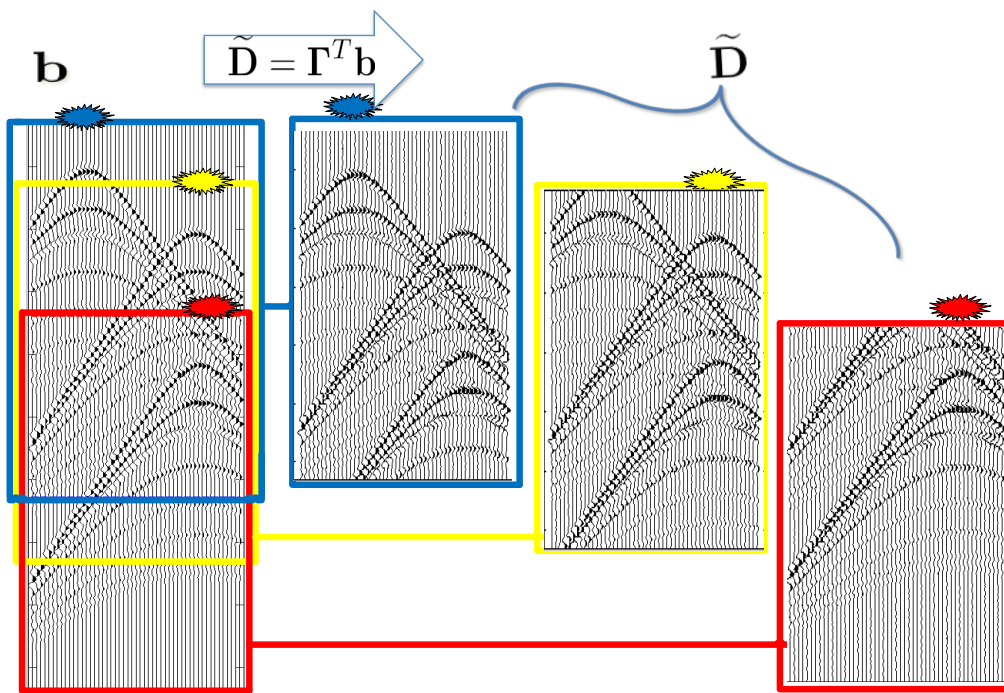


Figure 1 Schematic for the pseudodeblending operator.

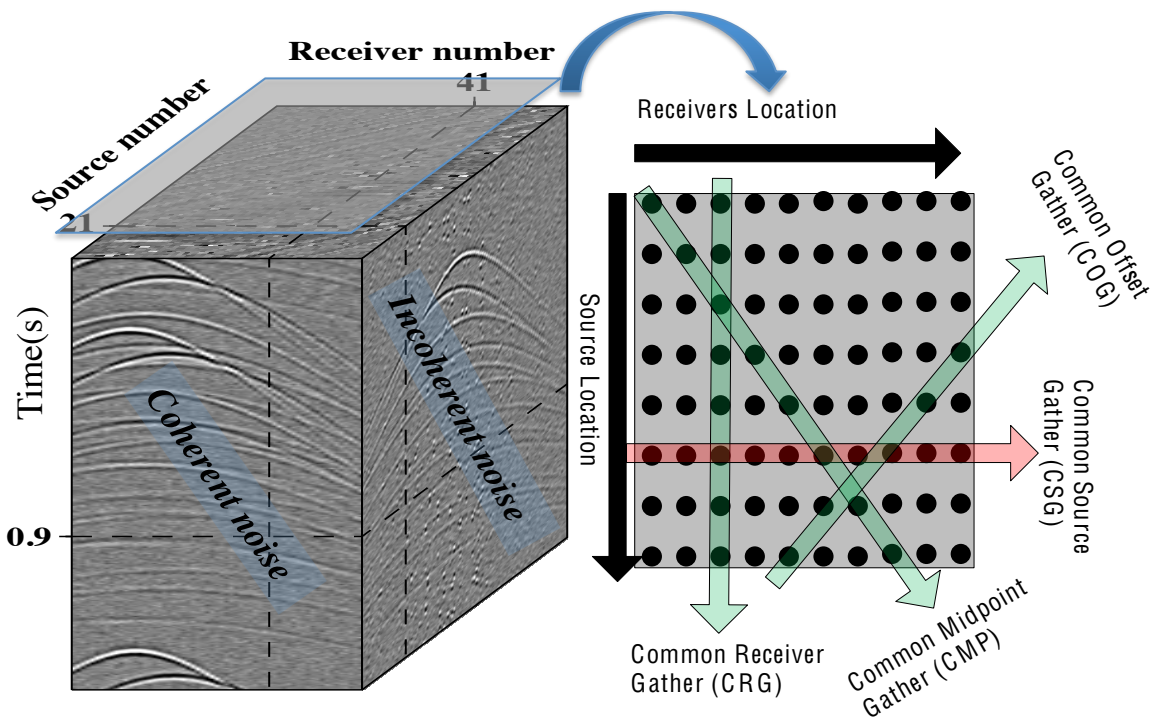
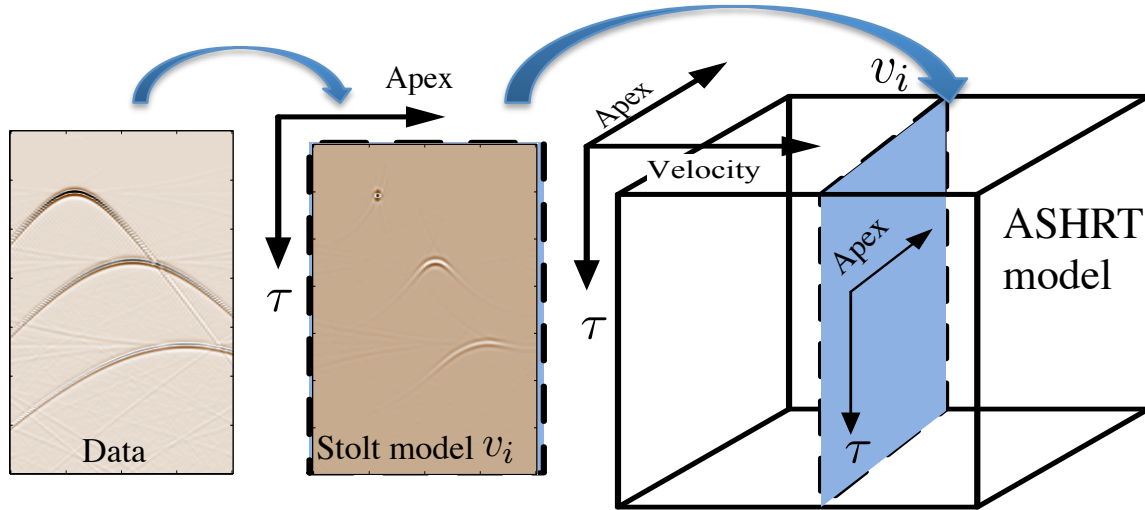


Figure 2 Pseudodeblended data cube and the different seismic gathers that can be used for denoising (show in in green arrow).



**Figure 3** Stolt operator acting as an alternative to apex shifted hyperbolic Radon operator.

Stolt operator estimates a cross-section in the ASHRT model cube. The ASHRT operator has a computational cost of  $O(n_a \times n_\tau \times n_v \times n_x)$ , where  $n_a, n_\tau, n_v$  and  $n_x$  are the numbers of apex locations, apex times, velocities and offsets, respectively. Assuming that we scan for all possible apex locations and times, then we can assume that  $n_a = n_x$  and  $n_\tau = n_t$ . Therefore the classical ASHRT cost is  $O(n_x^2 \times n_t \times n_v)$ . On the other hand, Stolt operator has a cost (with no zero padding) that is of the 2D FFT of the data with size  $n_t \times n_x$  followed by  $f-k$  mapping and inverse 2D FFT of the model with size  $n_t \times n_v \times n_x$ . Therefore, the total computational cost of an ASHRT implemented via Stolt migration/de-migration is  $O([n_t \log_2(n_t) + n_x \log_2(n_x)][n_v + 1] + n_v \times n_{kx} \times n_\omega)$ , where  $n_{kx}$  and  $n_\omega$  are number of horizontal wavenumbers and temporal frequencies, respectively. The cost of the  $f-k$  mapping is proportional to  $n_v \times n_{kx} \times n_\omega$  and we stress that the latter is an upper limit, since in practice we only scan for a limited band of positive frequencies and use the Fourier domain symmetry to compute the negative frequencies. Figure 4a shows the computational times of ASHRT and Stolt operator with and without zero padding. Zero padding is sometimes required to reduce artifacts associated with  $f-k$  interpolation. Figure 4b shows the improvement in the computational time of Stolt with and without zero padding compared to ASHRT. It is clear that an implementation of the ASHRT via Stolt operators can lead to a significant saving in computational costs. This is very important for processing large data set with a large number of pseudodeblended cubes.

## Inversion

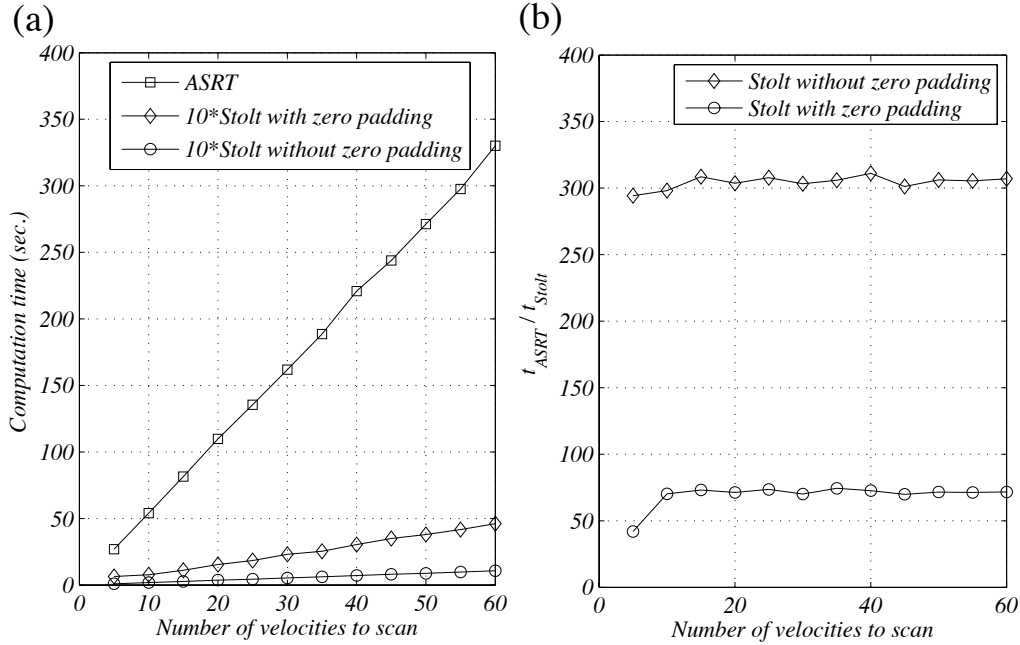
We assume that the data are contaminated with noise and therefore we pose the estimation of  $\mathbf{m}$  via the minimization of the vector of residuals

$$\mathbf{r} = \mathbf{d} - \mathbf{L}\mathbf{m}. \quad (5)$$

This is an ill-posed problem and, therefore, a regularization term must be included to estimate a unique and stable model  $\mathbf{m}$ . For example, the  $\ell_2$  regularization term results in smooth estimates of  $\mathbf{m}$ . On the other hand, and  $\ell_1$  regularization term induces solutions that are sparse. The inversion problem can be formulated by minimizing the following cost function

$$\begin{aligned} J &= \|\mathbf{r}\|_p^p + \mu \|\mathbf{m}\|_q^q \\ &= \|\mathbf{d} - \mathbf{L}\mathbf{m}\|_p^p + \mu \|\mathbf{m}\|_q^q. \end{aligned} \quad (6)$$

where the first term on the right hand side is the misfit term and the second term is the regularization term. In both terms we have assumed  $\ell_p$  and  $\ell_q$  norms are given by the general expressions  $\ell_p = \sum_i |r_i|^p$  and  $\ell_q = \sum_i |m_i|^q$ . By minimizing the cost function with respect to the unknown vector of Radon coefficients  $\mathbf{m}$  one finds a solution that honours the observations  $\mathbf{d}$ . The parameters  $p$  and  $q$  represent the exponent of the  $p$ -norm of the misfit and the  $q$ -norm of the model regularization term, respectively. Claerbout and Muir (1973) proposed using  $p = 1$  when the data is contaminated with erratic noise to estimate model that is robust. Since the Radon model is expected to be sparse, we can also use  $q = 1$  to estimate a sparse model (Sacchi and Ulrych, 1995; Trad et al., 2003).



**Figure 4** Comparing operators (a) Computation times. (b) Stolt computation times compared to ASHRT.

## Examples

We tested the robust Stolt-based ASHRT with one synthetic and one marine data set from the Gulf of Mexico. Both data sets are blended numerically with a 50% time reduction compared to the conventional acquisition. The blending scheme represents one source firing with random delays. The data are pseudodeblended into common receiver gathers to obtain Figures 5a and 6a. Stolt model estimated from the pseudodeblended common receiver gather via robust inversion scheme for each data set is shown in Figures 5b and 6b. The data recovered from the robust Radon models is shown in Figures 5c and 6c. The error of the estimated data is shown in Figures 5d and 6d. The quality of the recovered data is measured using the following expression

$$Q = 10 \text{Log} \frac{\|\mathbf{d}_{original}\|_2^2}{\|\mathbf{d}_{original} - \mathbf{d}_{recovered}\|_2^2}. \quad (7)$$

The  $Q$  values for the recovered synthetic data common receiver gather is 25.33 dB and for the real data common receiver gather is 11.55 dB.

## Conclusions

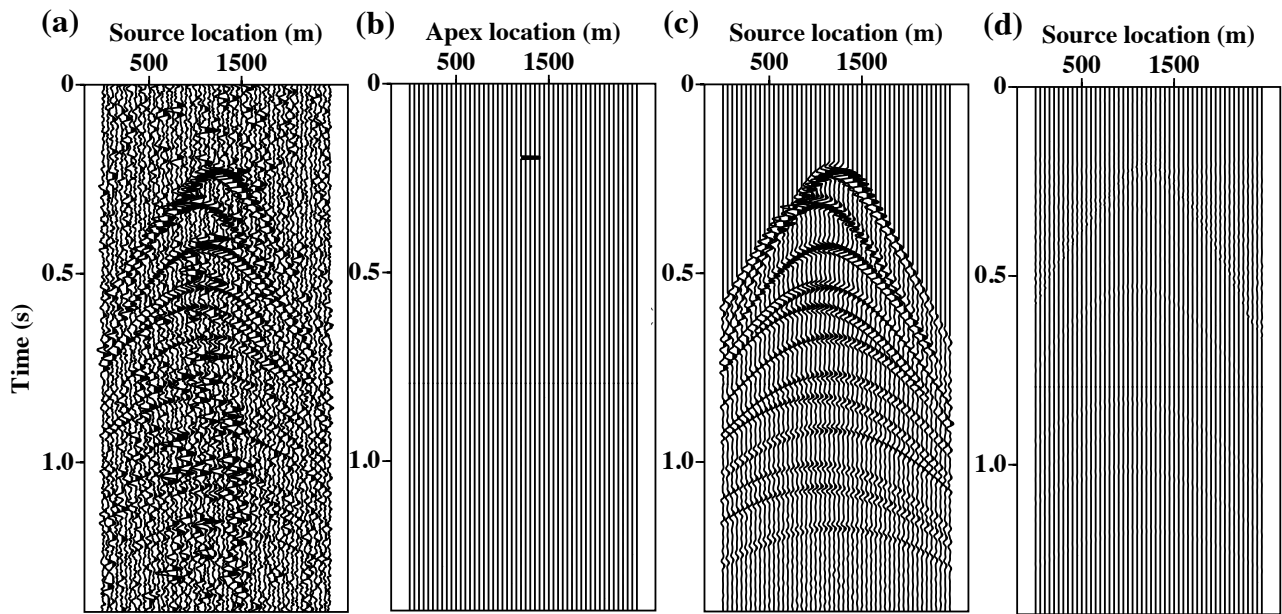
We have implemented Stolt operator to eliminate erratic incoherent noise that arises in common receiver gathers of blended sources data. We showed that source interferences in common receiver gathers could be removed by Stolt operator. Stolt operator is a more computationally efficient approach to apex shifted Radon transform. Since Stolt operator is implemented in  $f - k$  domain, it can be used in combination of non-uniform Fourier transform to interpolate missing traces.

## Acknowledgements

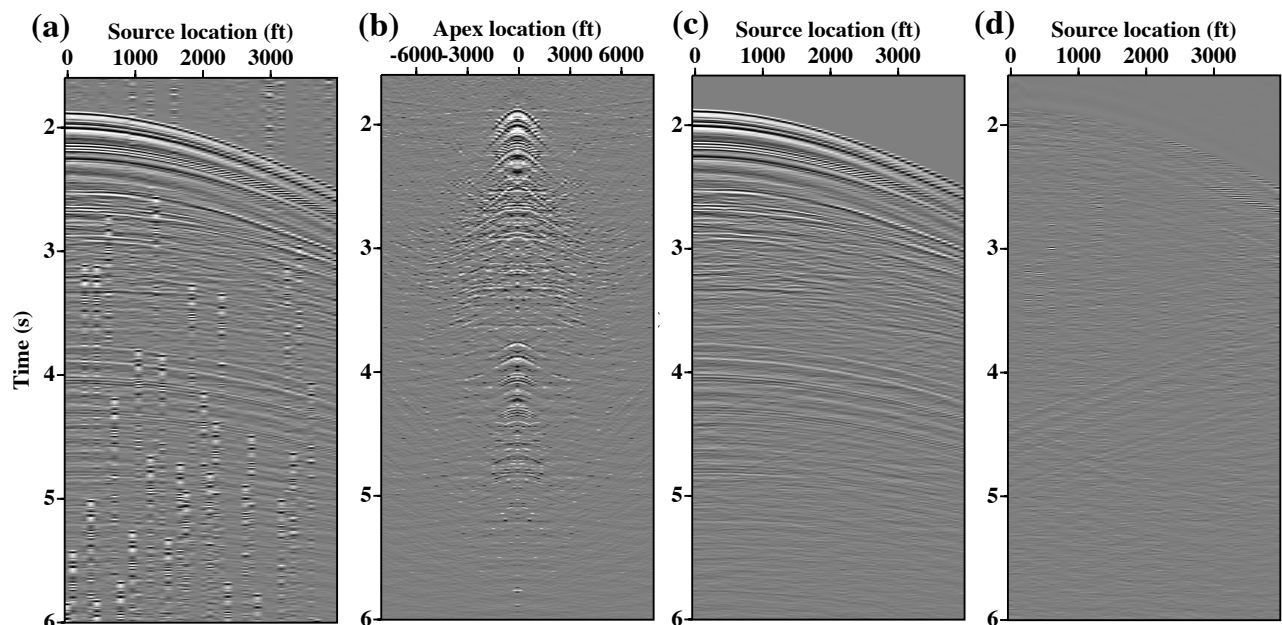
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**Figure 5** Synthetic data common receiver gather. (a) Pseudodeblended gather. (a) Stolt model for one velocity estimated using  $p = 1, q = 1$  inversion. (c) Data recovered by forward modelling  $p = 1, q = 1$  estimated model. (d) Error in recovered data.



**Figure 6** Real data common receiver gather. (a) Pseudodeblended gather. (a) Stolt model for one velocity estimated using  $p = 1, q = 1$  inversion. (c) Data recovered by forward modelling  $p = 1, q = 1$  estimated model. (d) Error in recovered data.

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