Converted wave shot profile wave equation migration toolbox kit

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Summary

In this paper, I introduce a time extrapolator equivalent from the depth counterpart for shot profile wave equation migration (WEM) for converted waves. I also affirm a few useful tools that have been available for the P-wave shot profile one-way WEM to be used for converted waves. The tools are namely: excitation time imaging condition for simplicity, time-shift imaging condition for massive speed gain, and topography consideration.

Introduction

Converted wave processing has gained in popularity over the last decade. Conventional processing involves a common conversion point (CCP) stacking followed by a converted wave poststack migration. In doing so, there are two limitation issues: firstly, CCP stacking assumes flat layered geologies, and secondly, post stack converted wave migration lumps the downgoing P-wave and upward going converted S-wave together which is just an approximation.

A better way to handle the above two limitation issues is to perform a prestack migration. Ray based Kirchhoff migration has been used for converted wave prestack time migration (CPSTM) (Cary, 2010) at least in the majority of Canadian data. But the Kirchhoff CPSTM operator also commits to flat geology CCP even though it does handle the P-wave and converted S-wave separately. Here, I propose to use a wave based converted wave shot profile WEM to easily handle both limitation issues simultaneously.

There are many advantages of using WEM: WEM by its nature exhibits all attributes of converted wave propagation phenomena; it honors the true CCP for all dipping geologies; there is no need for operator anti-aliasing filters as Kirchhoff migration requires; and it also preserves 'true' amplitude.

Methods

Let D be the P-wave downward propagating forward model of the source wavefield; initially D is unity at the surface $z=0$. Let U be the S-wave upward propagating receiver data wavefield that contains the observed converted wave shot record. Define W and its conjugate W^* as the wavefield extrapolators of any choice, where here we use a phase shift plus interpolation (Gazdag and Sguazzero, 1984) one-way wave equation extrapolator. Then for one depth step Δz, the depth extrapolation of U and D become,

$$D(k_x, z+\Delta z, \omega) = D(k_x, z, \omega) \cdot W^*(v_p) \cdot \Delta z \;, \qquad (1)$$

$$U(k_x, z+\Delta z, \omega) = U(k_x, z, \omega) \cdot W(v_s) \cdot \Delta z \;, \qquad (2)$$

where v_p and v_s are the interval velocities of P-wave and S-wave at depth z respectively.

In order to obtain the time migration one time step $\Delta\tau$ that is equivalent to one depth step Δz, I propose a change of variables:

$$\Delta z = v_p \cdot \Delta\tau_{\mathrm{pp}}/2 = v_s \cdot \Delta\tau_{\mathrm{ss}}/2 = v_{ps} \cdot \Delta\tau_{\mathrm{ps}}/2 \quad , \qquad (3)$$

where $\Delta\tau$ with two subscripts indicates two-way time, and v_{ps} the interval velocity of the converted P-S wave at imaged time τ. Since one common converted time step is needed for both U and D, I substitute equation (3) into (1) and (2) in order to obtain the wave equation for CPSTM:

$$D(k_x, \tau_{ps} + \Delta\tau_{ps}, \omega) = D(k_x, \tau_{ps}, \omega) \cdot W^*(v_p) \cdot v_{ps} / 2 \cdot \Delta\tau_{ps} \quad , \qquad (4)$$

$$U(k_x, \tau_{ps} + \Delta\tau_{ps}, \omega) = U(k_x, \tau_{ps}, \omega) \cdot W(v_s) \cdot v_{ps} / 2 \cdot \Delta\tau_{ps} \quad . \qquad (5)$$

The conventional imaging condition I is applicable for converted waves,

$$I(x,z,\omega) = U(x,z,\omega) \cdot D^*(x,z,\omega) / (D \cdot D^*) \ . \qquad (6)$$

There are a few useful tools used in P-wave shot profile WEM that can also be readily applied to the converted wave shot profile WEM to obtain the following benefits:

1. Speed up the migration by two times by using the excitation imaging condition (equation (8) of Ng, 1994).

   For time migration accuracy, the D need not to be extrapolated. Instead of using equation (6), just use the P-wave direct arrival travel time calculated from the source position to the subsurface image point to extract an output sample from the S-wave U wavefield. It is essentially the Green's function of the source.

2. Speed up the migration by two to five times by using time-shift imaging condition (Ng, 2007, 2008, 2009).

   This is essentially for depth migration usage where the U and D wavefields are extrapolated in coarser depth steps than normal depth size in order to gain speed. The zero lag time imaging condition in equation (6) does not provide any image between depth steps, and post imaging interpolation is a very difficult task. In order to obtain a good image between depth steps, it has been modified to a time-shift imaging condition with variable time lags. It uses a variation of the original Sava et al. (2003, 2005) and Rosales et al. (2008) time-shift imaging condition. Sava applies time- and space-shift imaging conditions to obtain the subsurface image angle gathers, and to perform velocity analysis while I use it for wavefield interpolation, a completely different application and benefit.

3. Topography consideration.

   For prestack depth shot profile WEM, separate treatment of the surface in the source side D and receiver side U are needed similar to that described in (Ng, 2008). But in prestack time shot profile WEM, regardless of how the surface is defined for velocity analysis, be consistent and use it, even if it is located at the asymptotic common conversion point (ACCP).

Example

Figure 1 shows an in-line section of a trace response of a 500 m offset 3D prestack time shot profile wave equation migration (WEM) for converted waves. Trace interval is 25 m. Variable velocities in time as well as variable $v_p v_s$ ratios were used. The bottom of the ellipses vary at different times showing the common conversion points (CCP) move towards the receiver side as time decreases. There is no operator aliasing observed as the WEM provides a natural nonaliased operator. The amplitude response is asymmetric, as is in P-wave shot domain migration. The phenomenon was first shown in Ng (1994, 1996) but without being fully understood until Zhang et al. (2003) gave an analytical explanation on ray density directivity.

Conclusions

Prestack time (or depth) shot profile wave equation migration can be used for converted waves with slight modifications - independent extrapolation using v_p and v_s on source and receiver wavefields. Since WEM produces much desired natural attributes, when it is used together with previous improved imaging schemes, it makes the shot profile WEM efficient as well as effective for converted wave migration.

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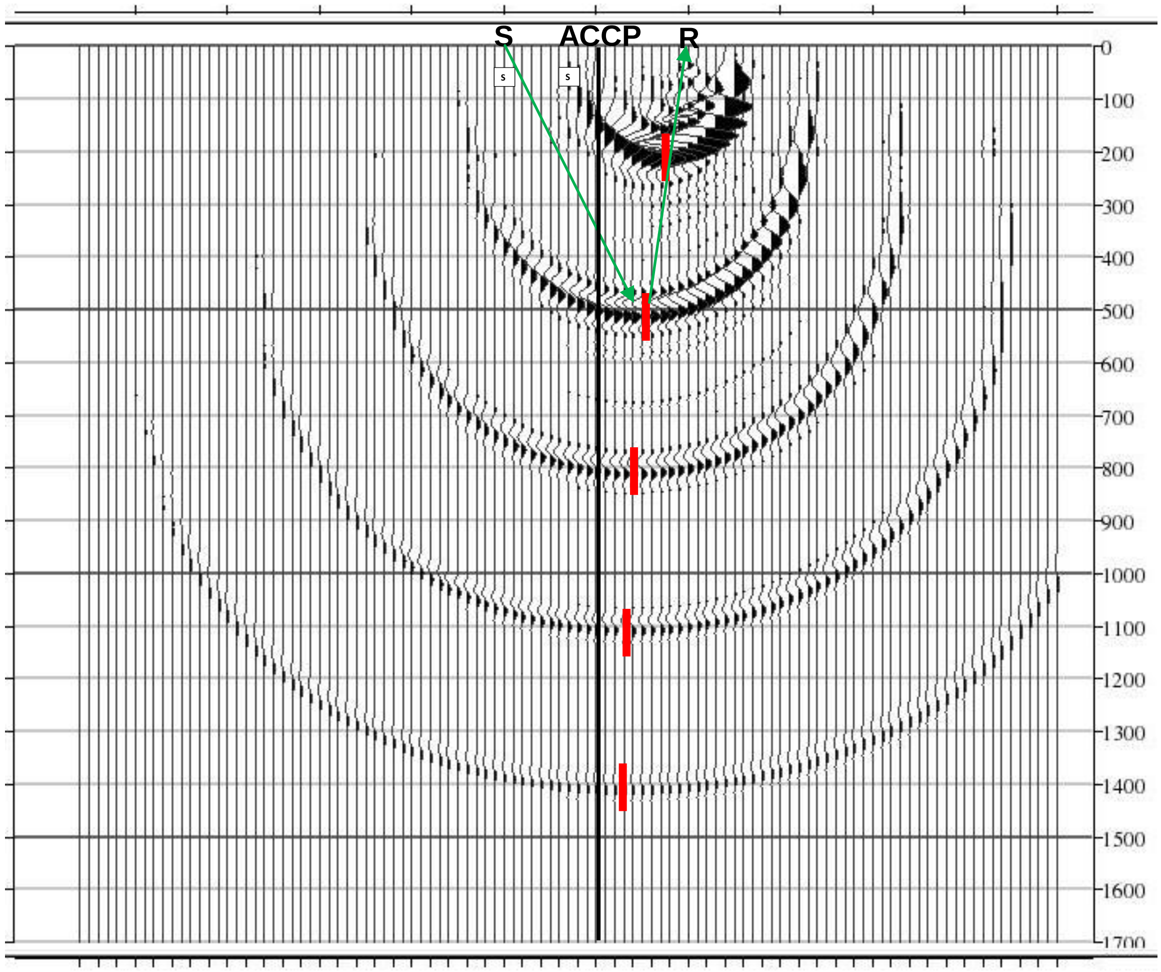

Figure 1. Nonzero offset trace response of the 3D shot profile WEM of converted waves. The red picks show the CCP at the bottom of the migration ellipses. The green arrows show the downward going P-wave of the source and upward going converted S-wave to the receiver. The asymmetric amplitude response is natural to shot profile migration.